On Renewable Sensor Networks with Wireless Energy Transfer

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Outline

- Introduction
- Problem description
- Optimal traveling path
- Results

Introduction (1/2)

- Envision employing a mobile vehicle (WCV) carrying a battery charging station to periodically visit each sensor node and charge it wirelessly.

- Introduce the concept of renewable energy cycle where the remaining energy level in a sensor node’s battery exhibit some periodicity over a time cycle.

Introduction (2/2)

- Formulate an optimization problem, with the objective of maximizing the ratio of the WCV's vacation time (time spent at its home service station) over the cycle time

- Develop a provable near-optimal solution for any desired level of accuracy.

Problem description (1/8)

- A set of sensor nodes N distributed over a two-dimensional area.
- Sensor node has a battery capacity of E_{max}
- Denote E_{min} the minimum energy at a sensor node battery.
- Each sensor node i sensing data with rate R_i

Problem description (2/8)

- Denote f_{ij} the flow rate from sensor node i to sensor node j (b/s)
- Denote f_{iB} the flow rate from sensor node i to the base station B (b/s)
- Following flow balance constraint at each sensor node i :

$$\sum_{k \in \mathcal{N}}^{k \neq i} f_{ki} + R_i = \sum_{j \in \mathcal{N}}^{j \neq i} f_{ij} + f_{iB} \quad (i \in \mathcal{N}). \quad (1)$$

Problem description (3/8)

- Energy consumption model

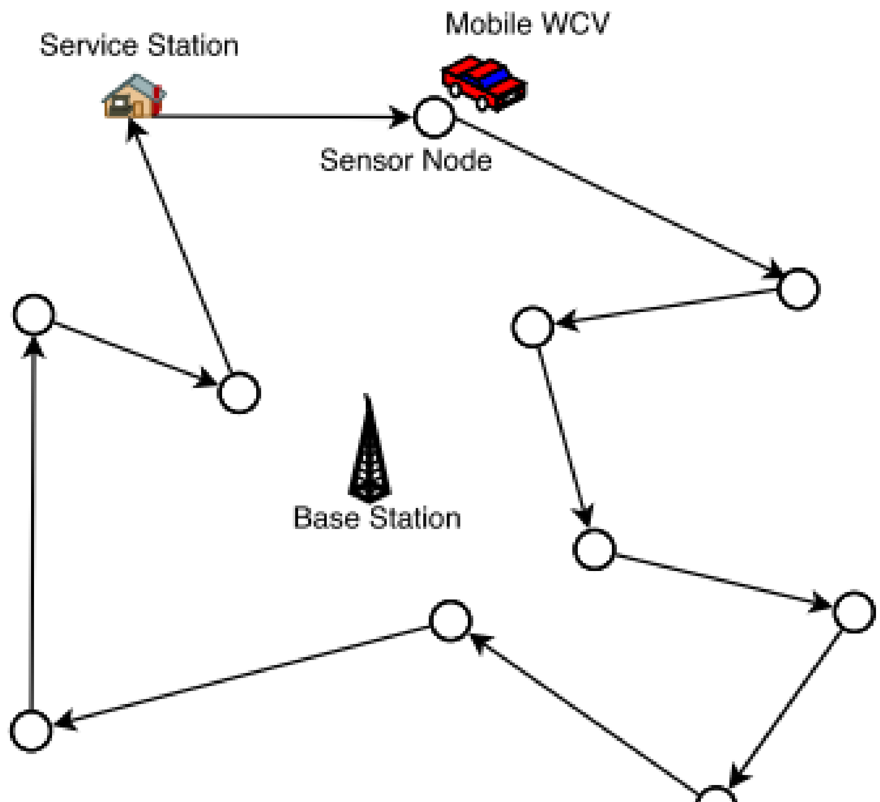
$$p_i = \rho \sum_{k \in \mathcal{N}}^{k \neq i} f_{ki} + \sum_{j \in \mathcal{N}}^{j \neq i} C_{ij} f_{ij} + C_{iB} f_{iB} \quad (i \in \mathcal{N}) \,, \quad (2)$$

  - ρ is the rate of energy consumption for receiving a unit of data rate
  - C_{ij} (or C_{iB}) is the rate of energy consumption for transmitting a unit of data rate from node i to node j (or the base station B)
  - $C_{ij} = \beta_1 + \beta_2 D_{ij}^{\alpha}$, D_{ij} is the distance between nodes, β_1 and β_2 are constants, α is the path loss index

Problem description (4/8)

- The traveling speed of the WCV is V (m/s)
- At a sensor node i, it will spend a time of τ_i to charge the sensor node
- Denote U the energy transfer rate of the WCV
- After the WCV visits all the sensor nodes, it will return to its service station, call this resting period vacation time, denoted as τ_{vac}

Problem description (5/8)



Problem description (6/8)

- Denote $P = (\pi_0, \pi_1, \ldots \pi_N, \pi_0)$ the path traversed by the WCV
- Denote a_i the arrival time of the WCV at sensor node i in the first renewable energy cycle: $$a_{\pi_i} = \tau + \sum_{k=0}^{i-1} \frac{D_{\pi_k \pi_{k+1}}}{V} + \sum_{k=1}^{i-1} \tau_k \ . \quad (3)$$
- The cycle time τ can be written as
$$\tau = \tau_P + \tau_{\text{vac}} + \sum_{i \in \mathcal{N}} \tau_i \ . \quad (4)$$
  - Denote D_p the distance of path P, $\tau_p = {}^{D_p}/_{V}$

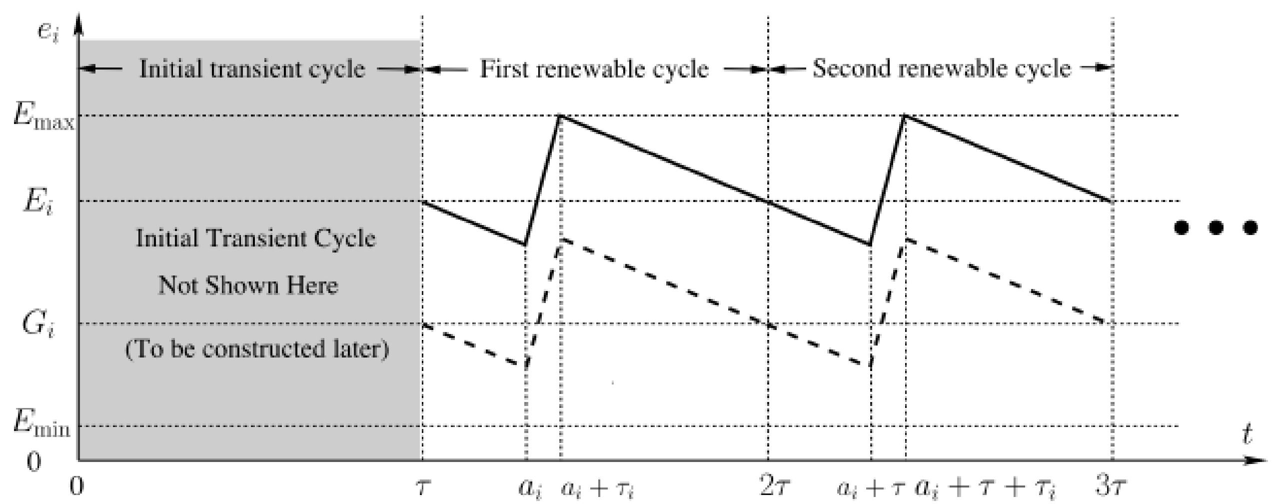
Problem description (7/8)

- During a renewable cycle, the amount of charged energy at a sensor node i during τ_i must be equal to the amount of energy consumed in the cycle:

$$\tau \cdot p_i = \tau_i \cdot U \quad (i \in \mathcal{N}) \,. \tag{5}$$

Problem description (8/8)

- $$e_i(a_i) = E_i - (a_i - \tau)p_i \geq E_{\min}.$$
- $$\begin{aligned} E_i = e_i(2\tau) &= e_i(a_i + \tau_i) - (2\tau - a_i - \tau_i)p_i \\ &= E_{\max} - (2\tau - a_i - \tau_i)p_i \ . \qquad (6) \end{aligned}$$
- $$E_{\max} - (\tau - \tau_i) \cdot p_i \geq E_{\min} \quad (i \in \mathcal{N}) \ . \qquad (7)$$



Optimal traveling path (1/6)

- *Theorem 1: In an optimal solution with the maximal* $\frac{\tau_{\text{vac}}}{\tau}$, *the WCV must move along the shortest Hamiltonian cycle that crosses all the sensor nodes and the service station.*

- Denote D_{TSP} as the total path distance in the shortest Hamiltonian cycle and $\tau_{TSP} = {}^{D_{TSP}}/_{V}$

- $\tau_{\text{TSP}} + \tau_{\text{vac}} + \sum_{i \in \mathcal{N}} \tau_i = \tau \ . \qquad (8)$

Optimal traveling path (2/6)

- **OPT**

$$\max \quad \frac{\tau_{\text{vac}}}{\tau}$$

$$s.t. \quad (1), (2), (5), (7), (8)$$

$$f_{ij}, f_{iB}, \tau_i, \tau, \tau_{\text{vac}}, p_i \geq 0 \quad (i, j \in \mathcal{N}, i \neq j)$$

- This problem has both nonlinear objective ($^{\tau\, vac}/_{\tau}$) and nonlinear terms (τp_i and $\tau_i\, p_i$) in constraints (5) and (7).

Optimal traveling path (3/6)

- For nonlinear objective ${}^{\tau_{vac}}/_{\tau}$:

$$\eta_{\mathrm{vac}} = \frac{\tau_{\mathrm{vac}}}{\tau} \; . \qquad (9)$$

- For (8), divide both sides by τ :

$$\tau_{\mathrm{TSP}} \cdot \frac{1}{\tau} + \eta_{\mathrm{vac}} + \sum_{i \in \mathcal{N}} \frac{\tau_i}{\tau} = 1.$$

- $$\eta_i = \frac{\tau_i}{\tau} \quad (i \in \mathcal{N}) \; , \qquad (10)$$

$$h = \frac{1}{\tau} \; . \qquad (11)$$

- $$h = \frac{1 - \sum_{i \in \mathcal{N}} \eta_i - \eta_{\mathrm{vac}}}{\tau_{\mathrm{TSP}}} \; . \qquad (12)$$

Optimal traveling path (4/6)

- **OPT-R**

$$\max \quad \eta_{\text{vac}}$$

$$s.t. \quad \sum_{j\in\mathcal{N}}^{j\neq i} f_{ij} + f_{iB} - \sum_{k\in\mathcal{N}}^{k\neq i} f_{ki} = R_i \qquad (i \in \mathcal{N}) \quad (15)$$

$$C\sum_{k\in\mathcal{N}}^{k\neq i} f_{ki} + \sum_{j\in\mathcal{N}}^{j\neq i} C_{ij} f_{ij} + C_{iB} f_{iB} - U\eta_i = 0 \qquad (i \in \mathcal{N}) \quad (16)$$

$$\eta_{\text{vac}} \leq 1 - \sum_{k\in\mathcal{N}} \eta_k - \frac{U \cdot \tau_{\text{TSP}}}{E_{\max} - E_{\min}} \cdot \boxed{\eta_i \cdot (1 - \eta_i)} \qquad (i \in \mathcal{N}) \quad (17)$$

$$f_{ij}, f_{iB} \geq 0, 0 \leq \eta_i, \eta_{\text{vac}} \leq 1 \qquad (i, j \in \mathcal{N}, i \neq j)$$

Optimal traveling path (5/6)

- Piecewise Linear Approximation for η_i^2

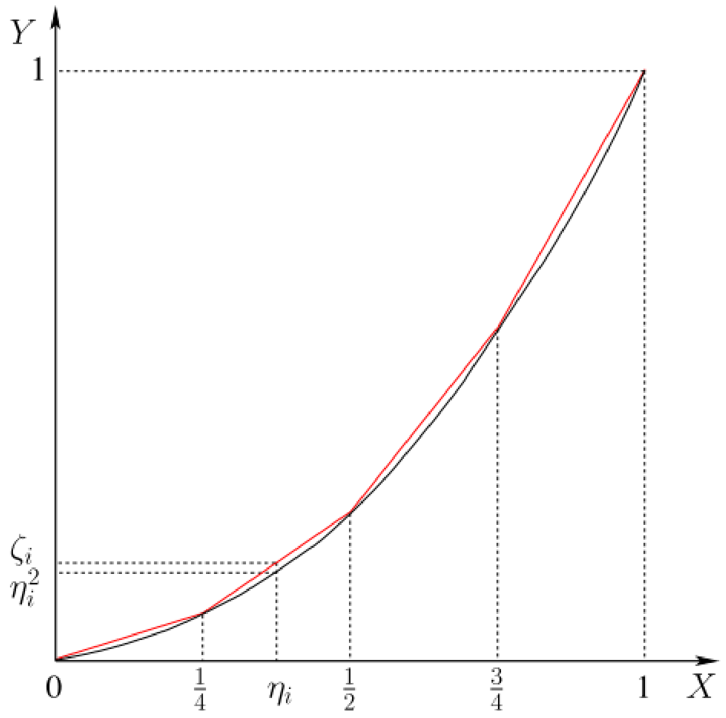


Fig. 3. An illustration of piecewise linear approximation (with $m = 4$) for the curve (η_i, η_i^2), $0 \leq \eta_i \leq 1$.

Optimal traveling path (6/6)

Construction of a Near-Optimal Solution

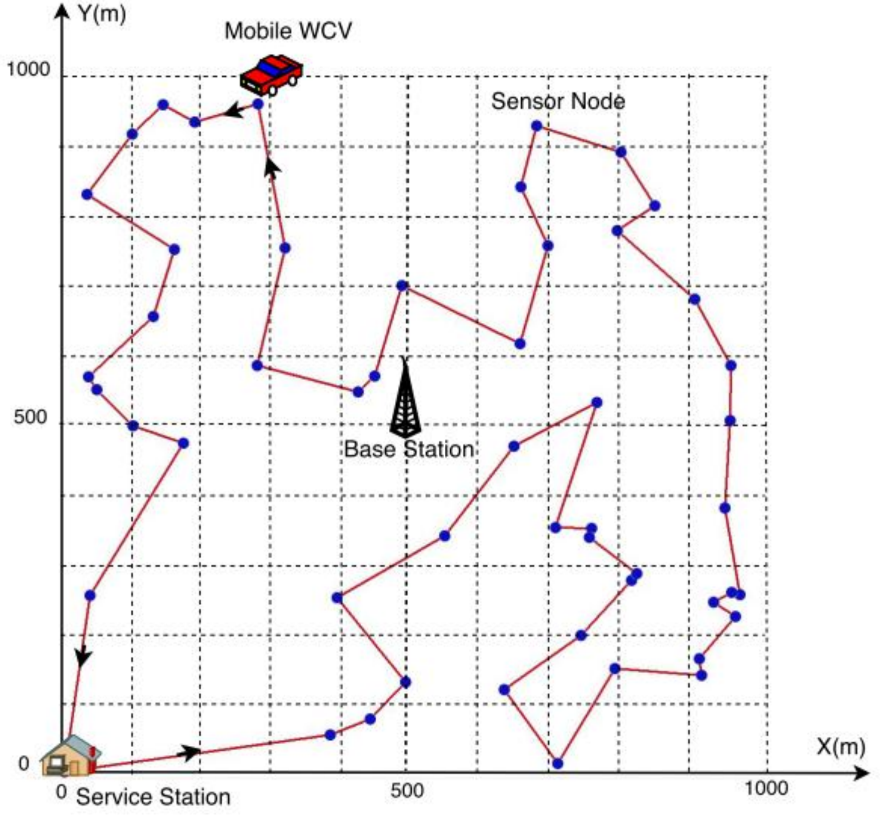
1. Given a target performance gap ϵ.
2. Let $m = \left\lceil \sqrt{\frac{U\tau_{\mathrm{TSP}}}{4\epsilon(E_{\max} - E_{\min})}} \right\rceil$.
3. Solve problem OPT-L with m segments by CPLEX, and obtain its solution $\hat{\psi} = (\hat{f}_{ij}, \hat{f}_{iB}, \hat{\eta}_i, \hat{\eta}_{\mathrm{vac}}, \hat{z}_{ik}, \hat{\lambda}_{ik}, \hat{\zeta}_i)$.
4. Construct a feasible solution $\psi = (f_{ij}, f_{iB}, \eta_i, \eta_{\mathrm{vac}})$ for problem OPT-R by letting $f_{ij} = \hat{f}_{ij}$, $f_{iB} = \hat{f}_{iB}$, $\eta_i = \hat{\eta}_i$ and $\eta_{\mathrm{vac}} = \min_{i \in \mathcal{N}} \{1 - \sum_{k \in \mathcal{N}} \hat{\eta}_k - \frac{U\tau_{\mathrm{TSP}}}{E_{\max} - E_{\min}} \cdot \hat{\eta}_i \cdot (1 - \hat{\eta}_i)\}$.
5. Obtain a near-optimal solution $(f_{ij}, f_{iB}, \tau, \tau_i, \tau_{\mathrm{vac}}, p_i)$ to problem OPT by Algorithm 1.

Results (1/3)

- Simulation Settings
  - Consider a randomly generated WSN consisting of 50 nodes
  - Sensor nodes over a square area of 1 km $\times$ 1 km
  - The traveling speed of the WCV is V= 5 m/s
  - Let E_{max}= 10.8 KJ , $E_{min} = 5.4$ KJ
  - U= 5W

Results (2/3)

- In this optimal cycle, D_{TSP}=5821 m and τ_{TSP}= 1164.2 sec
- For the target ϵ = 0.01, m= 4
- τ =17.34 hour, τ_{vac}=1164.2 sec and η_{vac}= 77.51%



Results (3/3)

- There exists a bottleneck node in the network with its energy dropping to E_{min} during a renewable energy cycle