Department of Computer Science
National Tsing Hua University

CS 5244: Introduction to Cyber Physical Systems

Unit 8: Extended State Machines and Hybrid Automata (Ch. 4)

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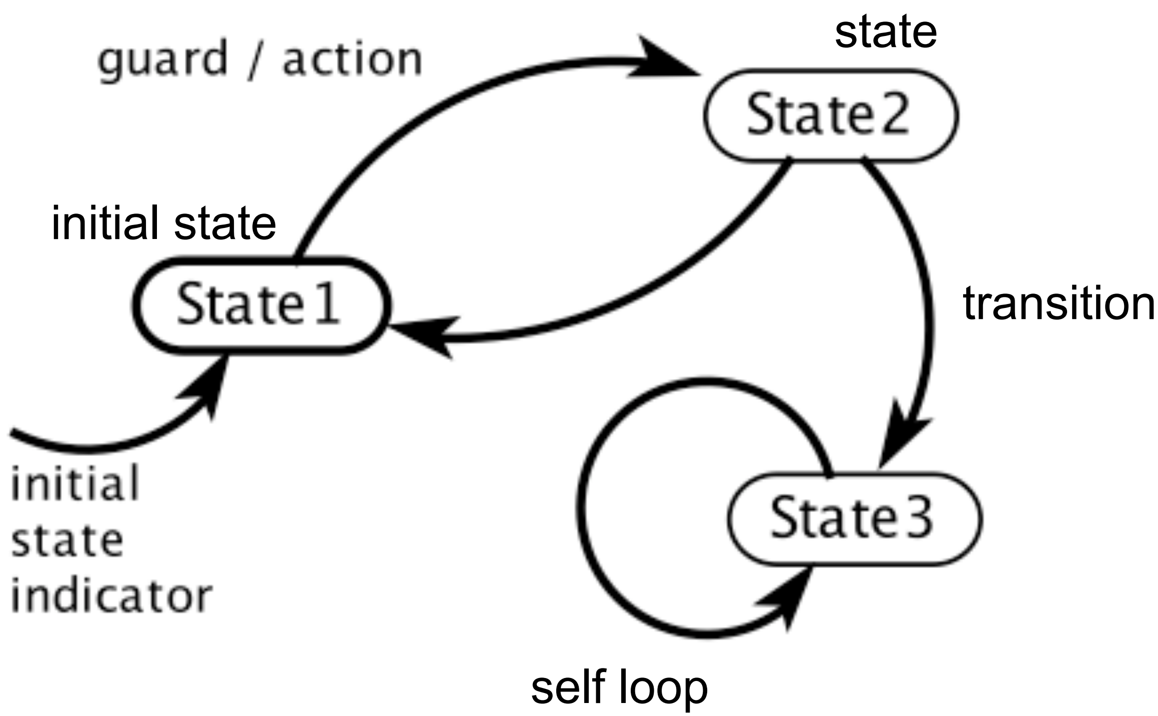
Acknowledgement: The instructor thanks Profs. Edward A. Lee & Sanjit A. Seshia at UC Berkeley for sharing their course materials

Modeling with State Machines

Questions:

- How to represent the system for:
  - Systematic analysis
  - So that a computer program can manipulate it
- How to model its environment
- How to compose subsystems to make bigger systems
- How to check whether the system satisfies its specification in its operating environment

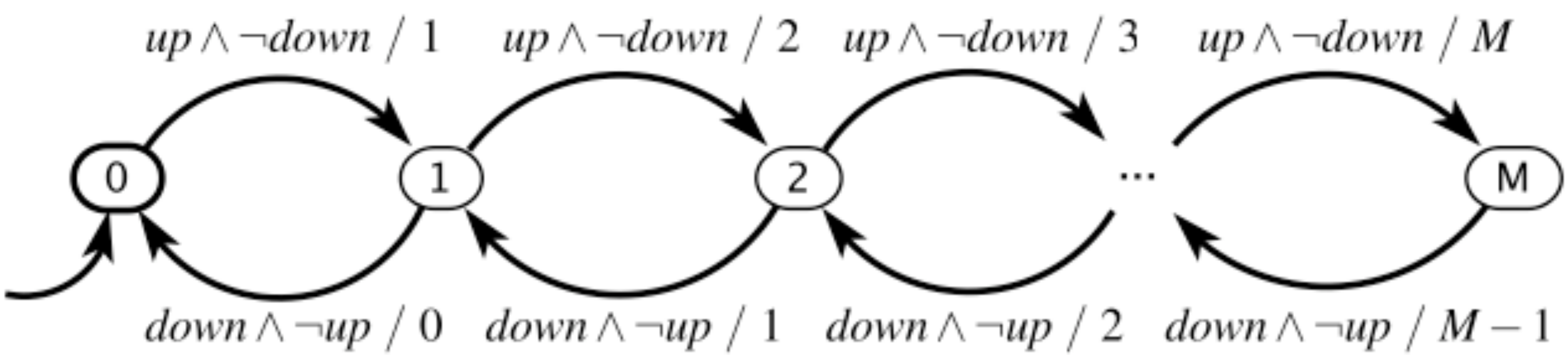
Recall FSM Notation



Garage Counter Example

Recall this example, which counts cars in a parking garage:

inputs: $up, down \in \{present, absent\}$
output $\in \{0, \cdots, M\}$

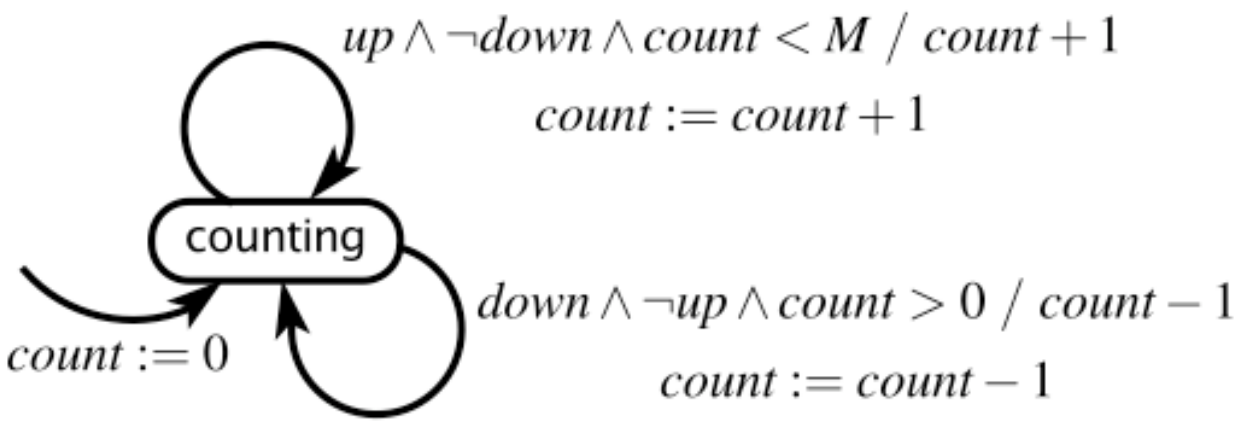


The notation here is a bit awkward, because the parameter *M* may be large, and we are stuck using a somewhat informal … notation.

Extended State Machines

Extended state machines augment the FSM model with *variables* that may be read or written. E.g.:

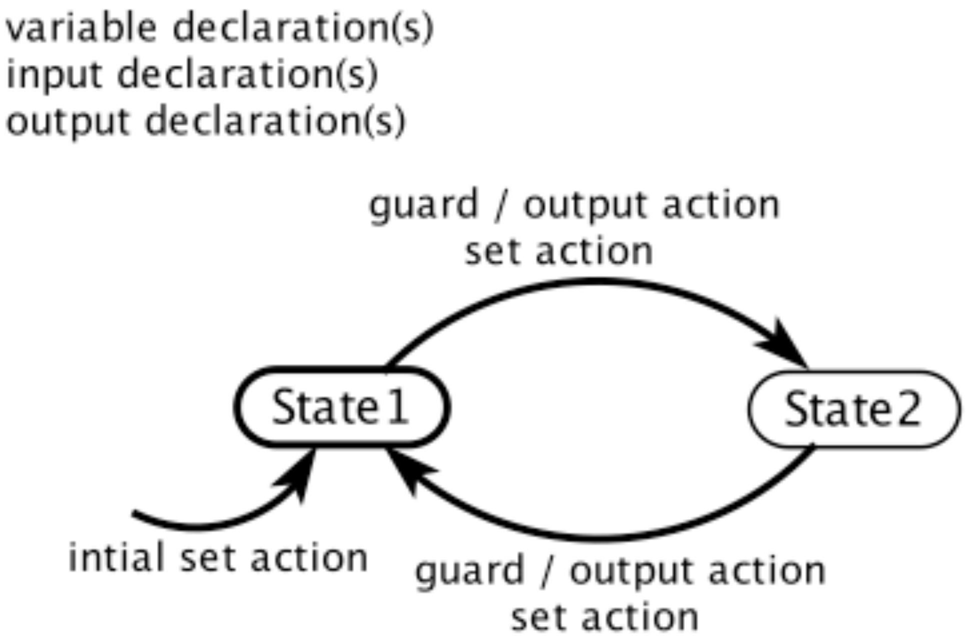
variable: $count \in \{0, \cdots, M\}$
inputs: $up, down \in \{present, absent\}$
output $\in \{0, \cdots, M\}$



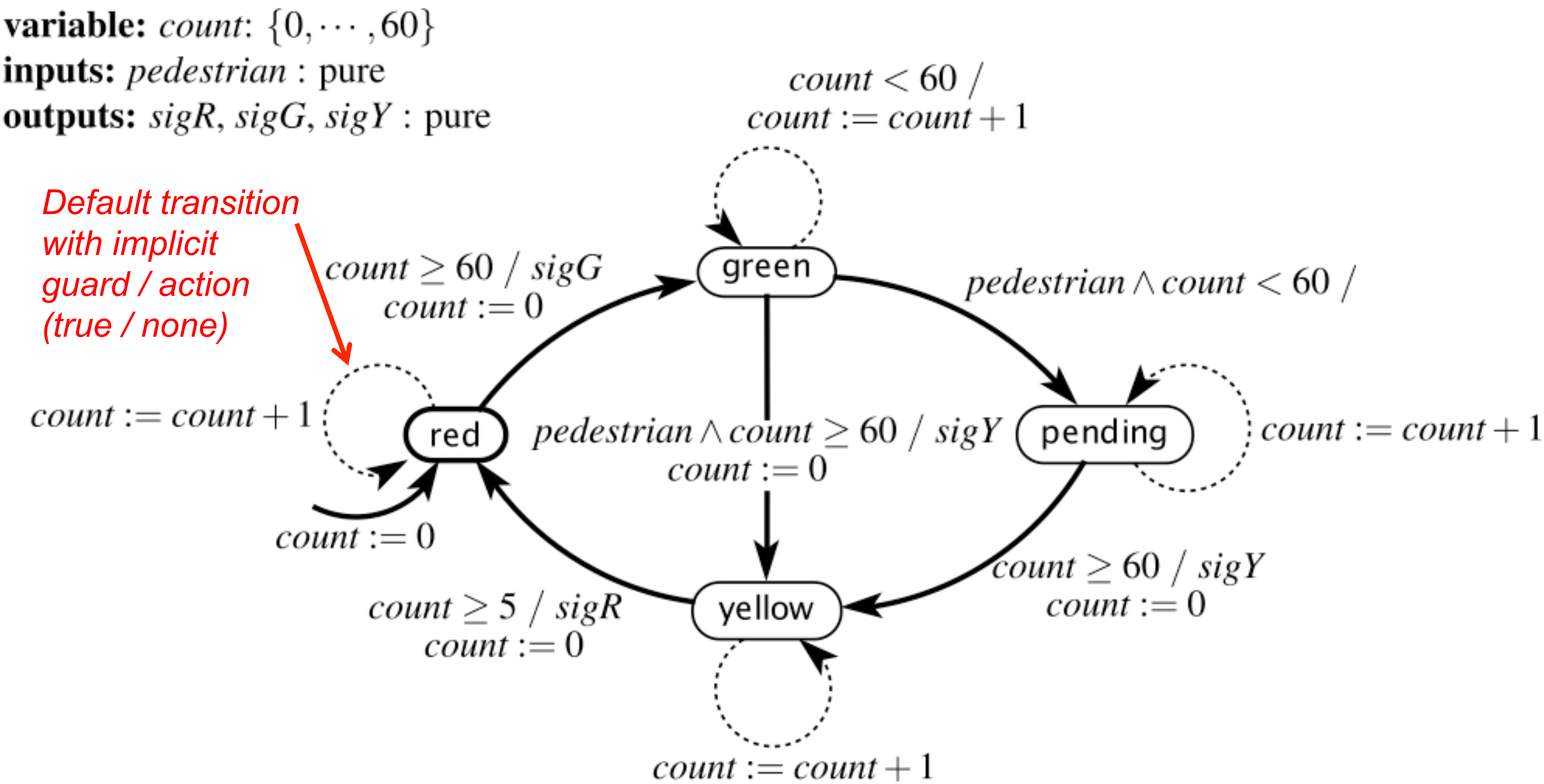
Question: What is the size of the state space?

General Notation for Extended State Machines

We make explicit declarations of variables, inputs, and outputs to help distinguish the three.

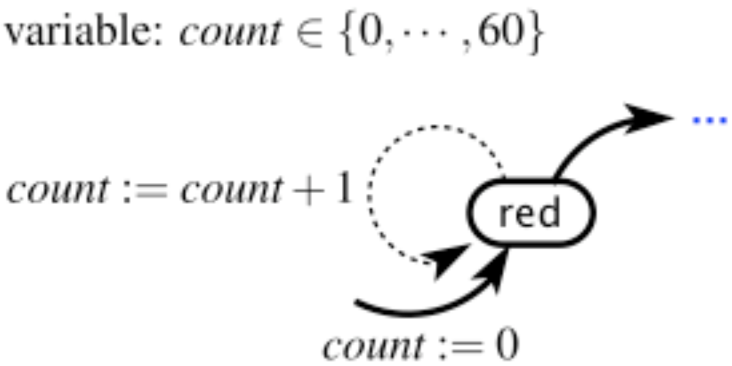


Extended state machine model of a traffic light controller at a pedestrian crossing:



This model assumes one reaction per second (a *time-triggered* model)

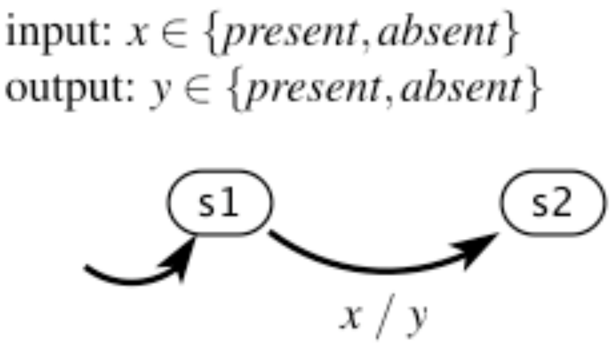
When does a reaction occur?



When a reaction occurs is not specified in the state machine itself. It is up to the environment.

This traffic light controller design assumes one reaction per second. This is a *time-triggered model*.

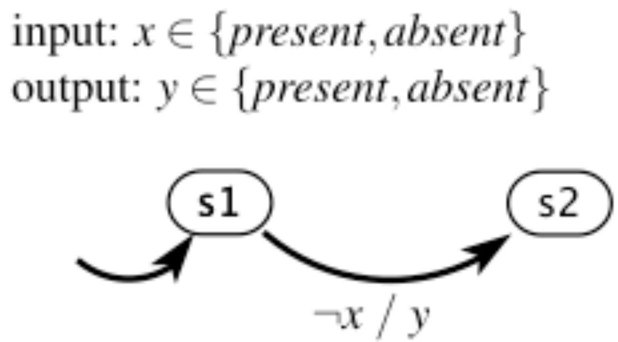
When does a reaction occur?



Suppose all inputs are discrete and a reaction occurs when any input is present. Then the above transition will be taken whenever the current state is s1 and *x* is present.

This is an *event-triggered model*.

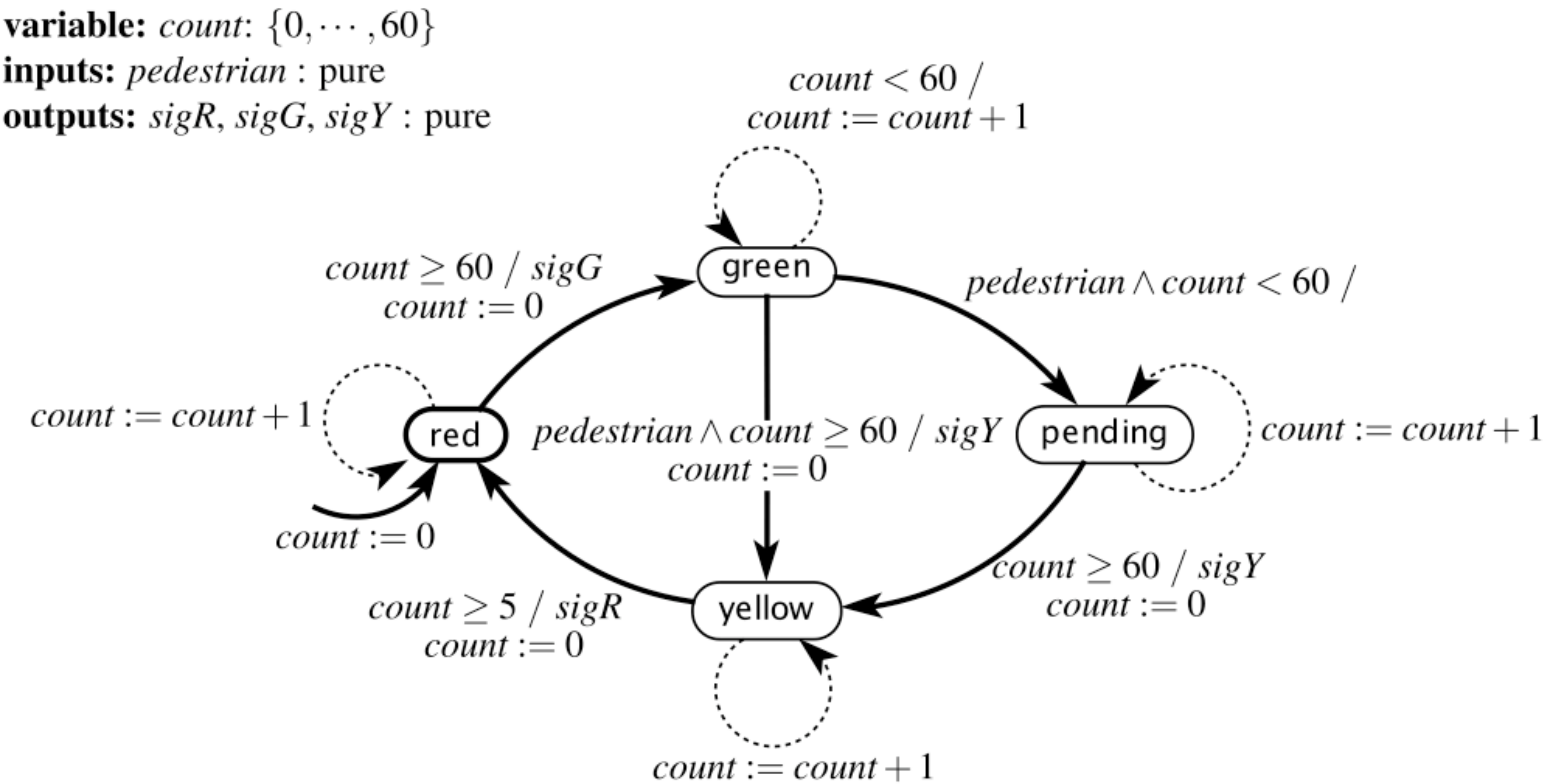
When does a reaction occur?



Suppose *x* and *y* are discrete and pure signals.
When does the transition occur?

Answer: when the environment triggers a reaction and x is absent. If this is a (complete) event-triggered model, then the transition will never be taken because the reaction will only occur when x is present!

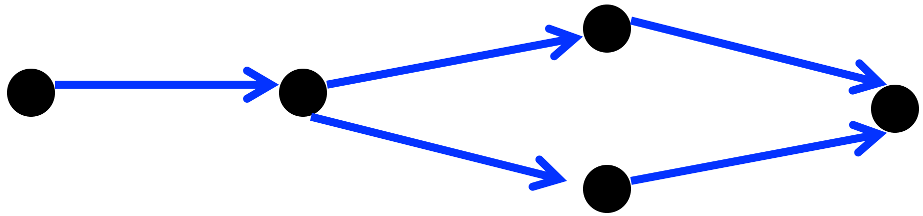
Recall: extended state machine model of a traffic light controller at a pedestrian crossing:



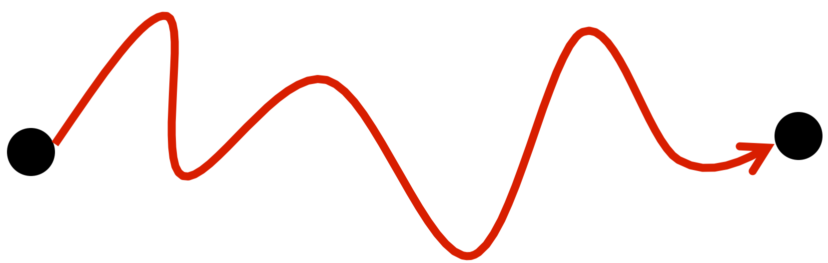
This is a time-triggered model that assumes one reaction per second.

Hybrid Automata

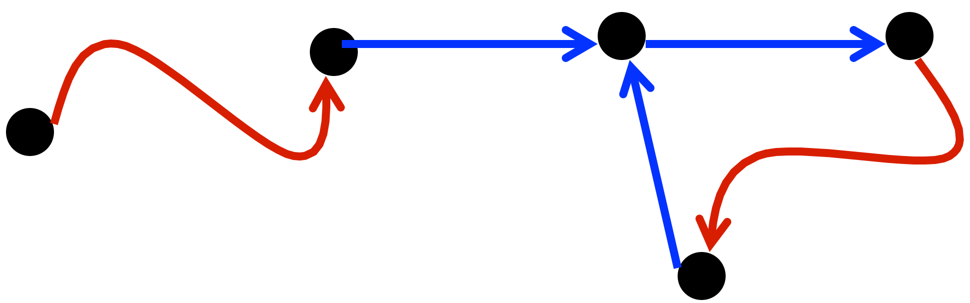
Discrete System (FSM)

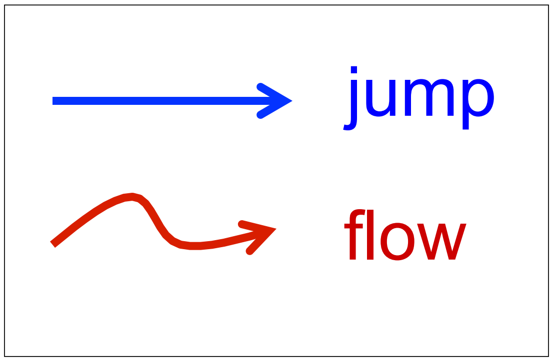


Continuous System



Hybrid System

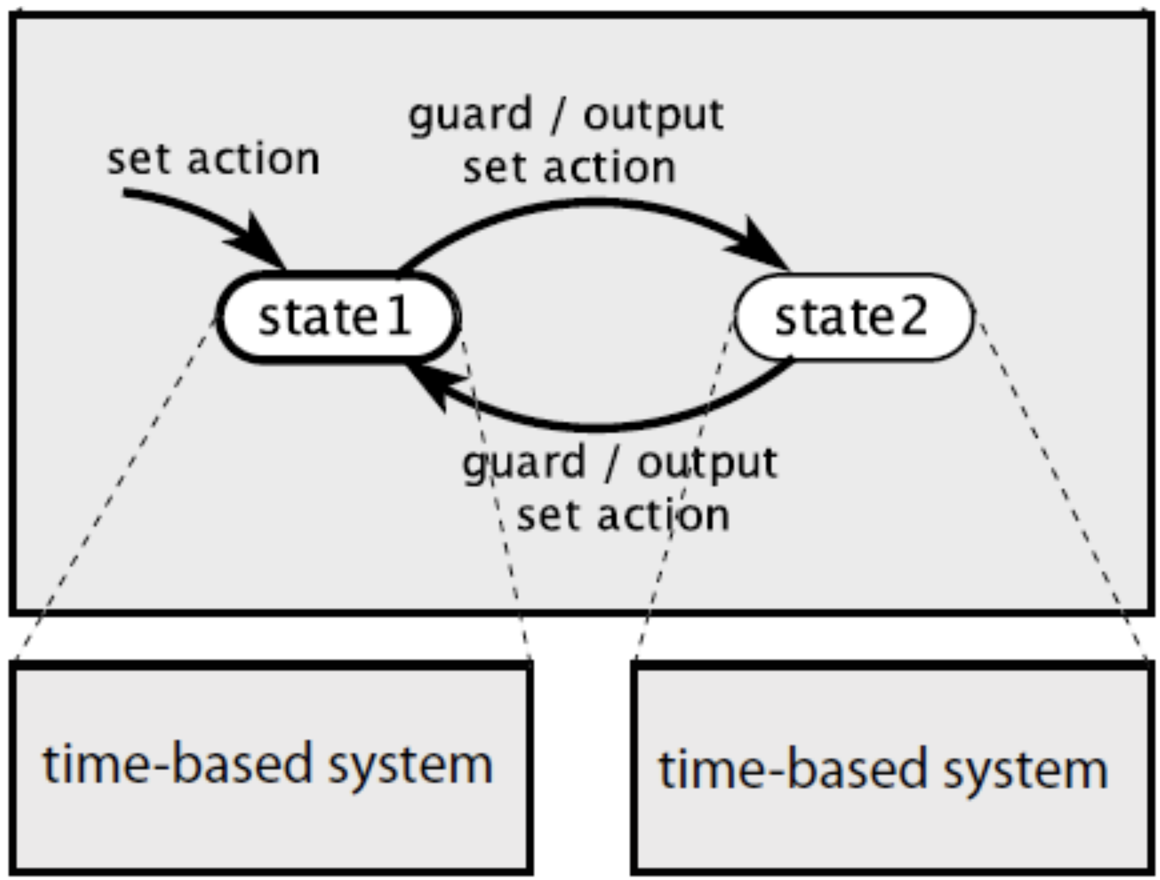




Where do Hybrid Systems arise?

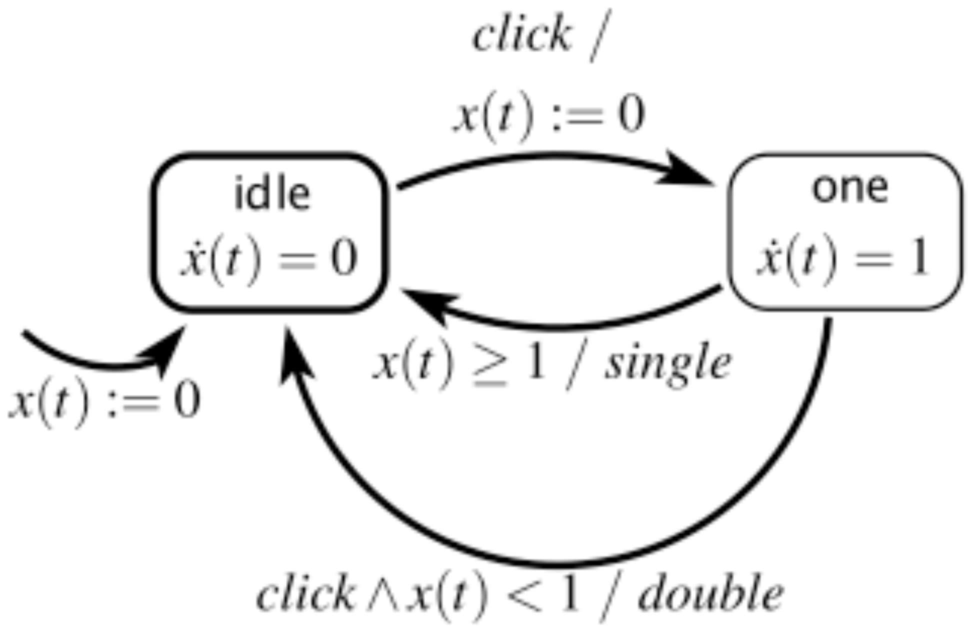
- Digital controller of physical “plant”
  - thermostat
  - intelligent cruise control in cars
  - aircraft auto pilot
- Phased operation of natural phenomena
  - bouncing ball
  - biological cell growth
- Multi-agent systems
  - ground and air transportation systems
  - interacting robots

An alternative to FSMs that is explicit about the passage of time: ***Timed automata***, a special case of hybrid systems.



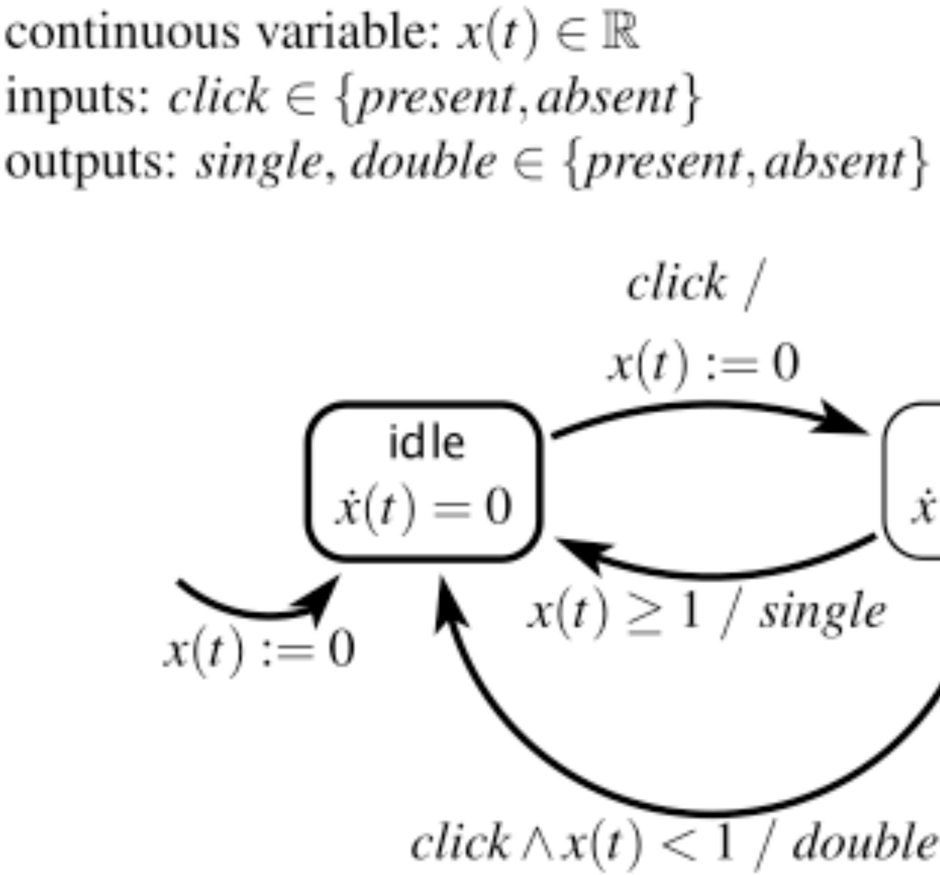
Example: Mouse Double Click Detector

continuous variable: $x(t) \in \mathbb{R}$
inputs: $click \in \{present, absent\}$
outputs: $single, double \in \{present, absent\}$



This simple form of hybrid system is called a *timed automaton*, where the dynamics is just passage of time.

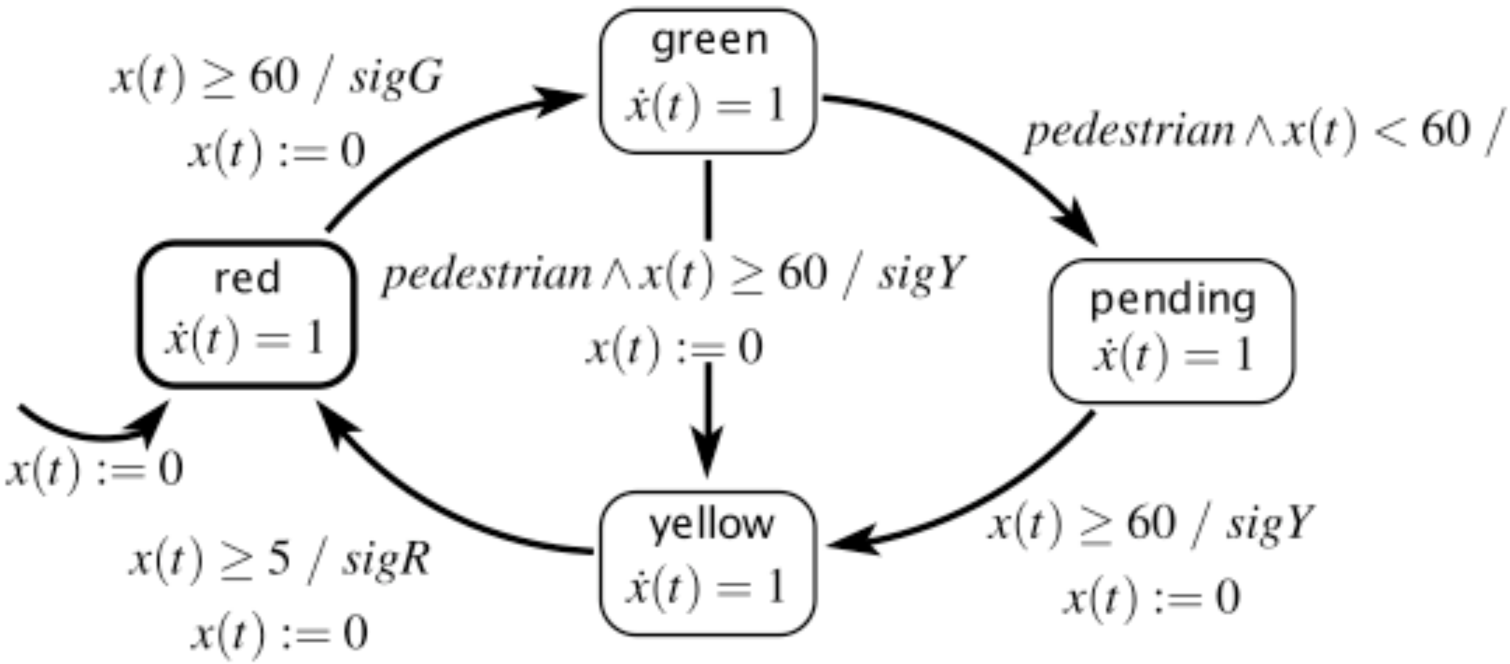
Example: Mouse Double Click Detector



How many states does this automaton have?

Timed automaton model of a traffic light controller

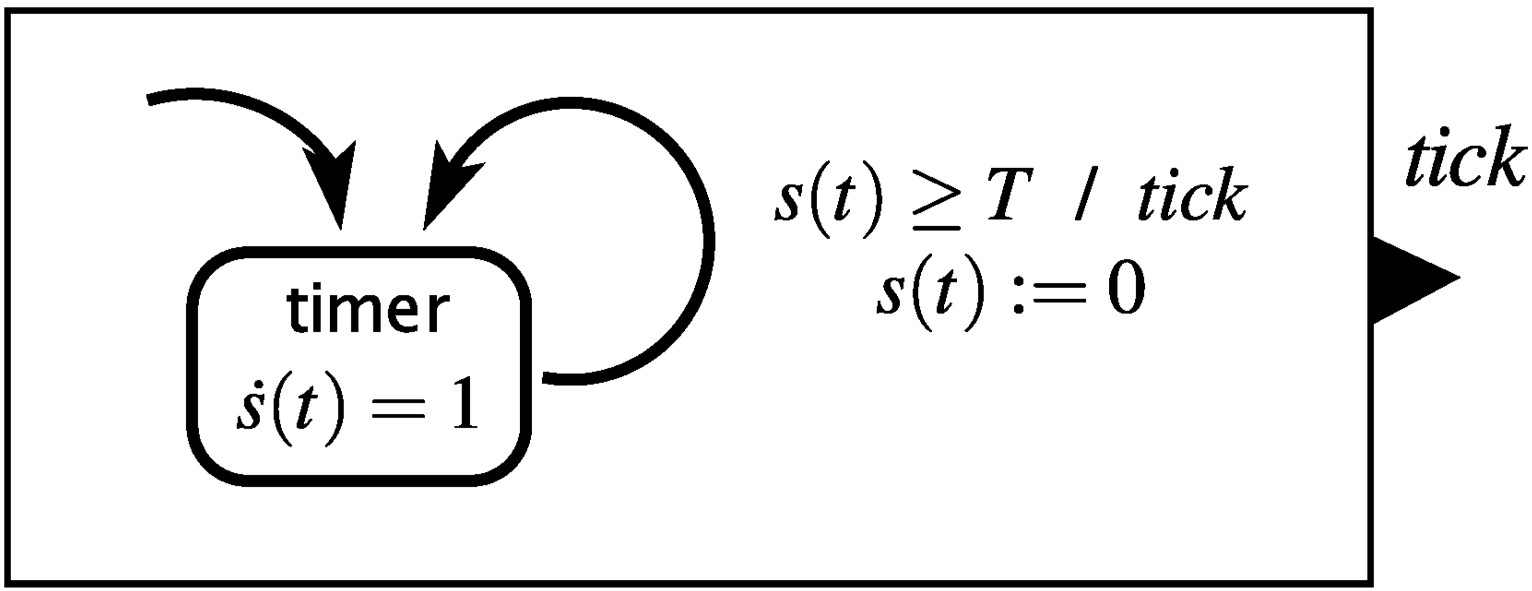
continuous variable: $x(t) : \mathbb{R}$
inputs: $pedestrian$: pure
outputs: $sigR, sigG, sigY$: pure



This light remains green at least 60 seconds, and then turns yellow if a pedestrian has requested a crossing. It then remains red for 60 seconds.

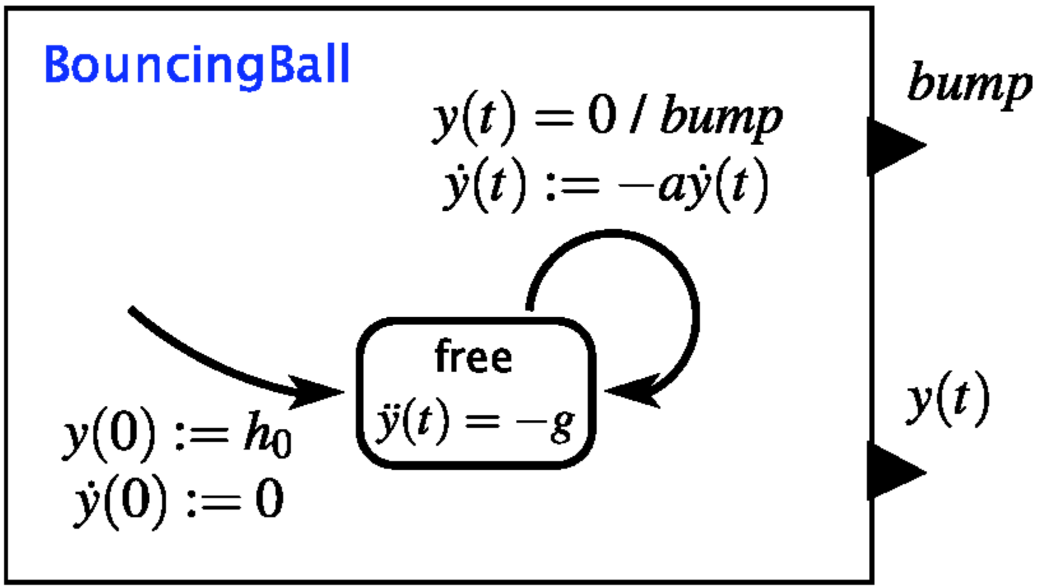
Example: "Tick" Generator (Timer)

How would you model a timer that generates a 'tick' each time T time units elapses?



A similar timed automaton can model a generator of a timer interrupt.

Hybrid Automaton for Bouncing Ball

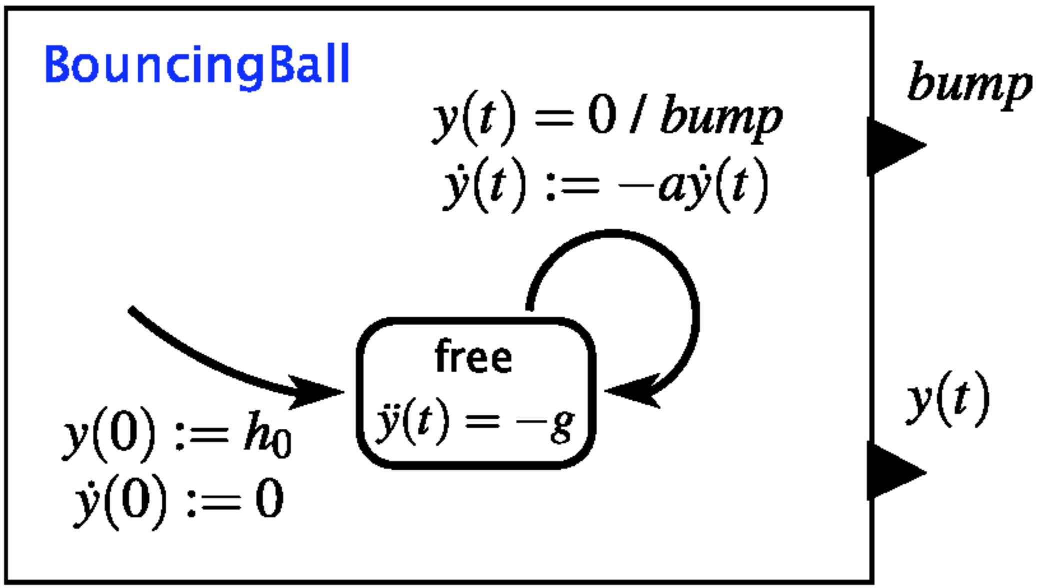


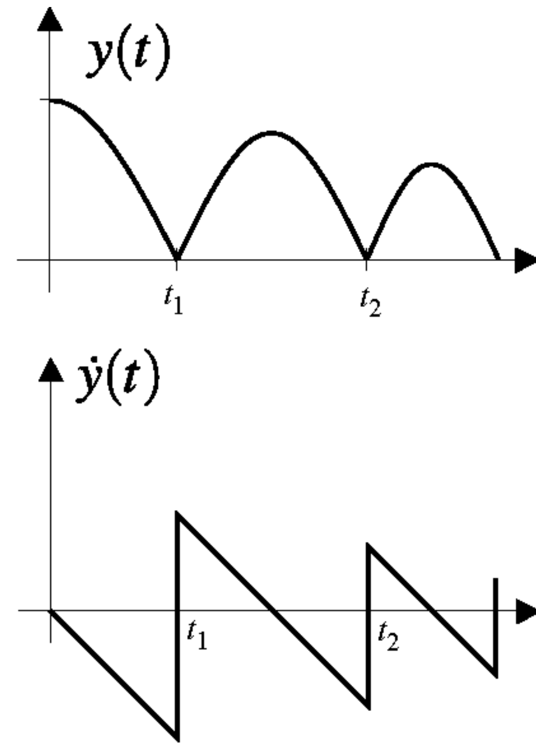
y – vertical distance from ground (position)

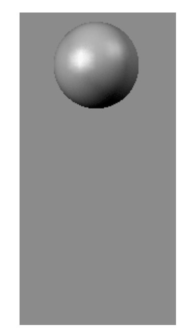
a – coefficient of restitution, $0 \cdot a \cdot 1$

If you plotted y(t), what would it look like?

Hybrid Automaton for Bouncing Ball





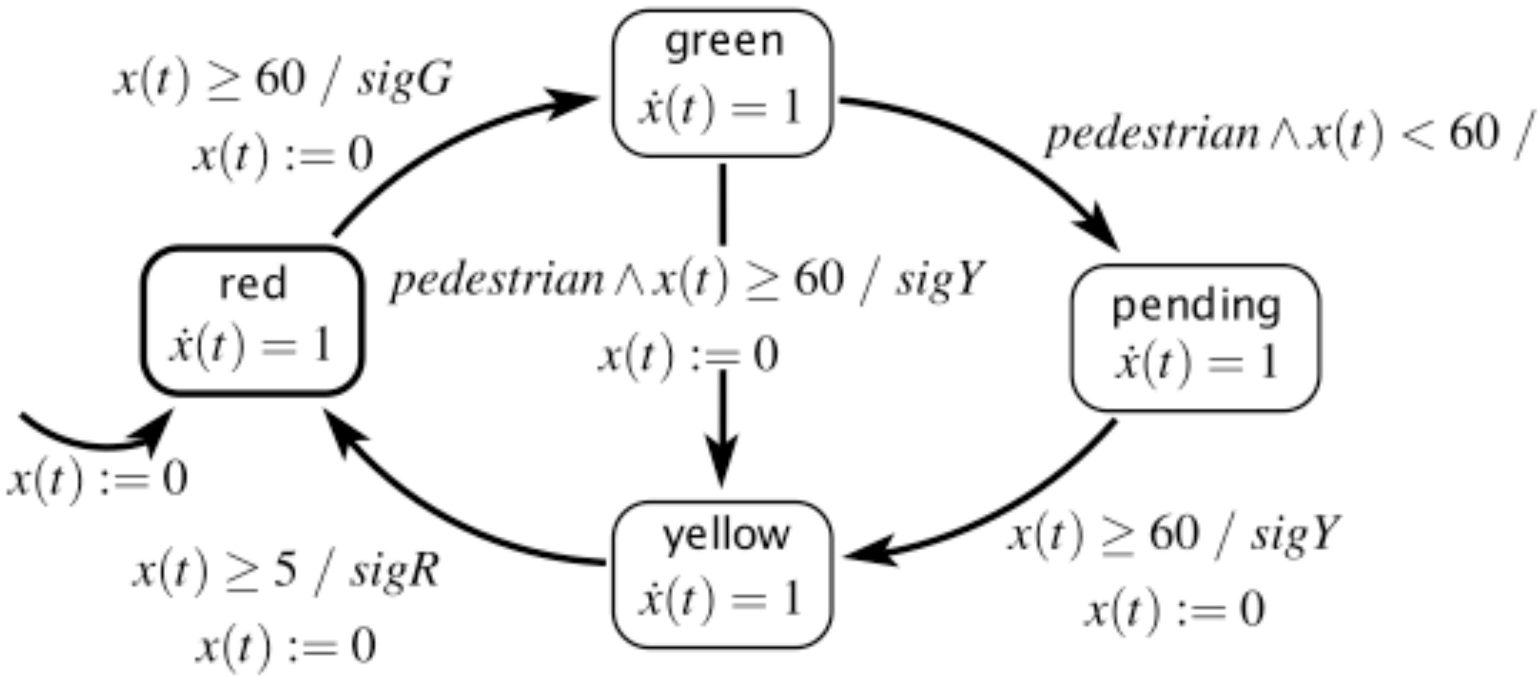


y – vertical distance from ground (position)

a – coefficient of restitution, $0 \cdot a \cdot 1$

When do reactions occur in a hybrid automaton?

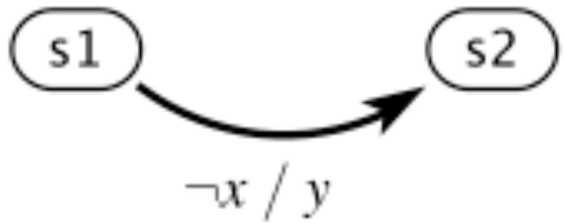
continuous variable: $x(t) : \mathbb{R}$
inputs: $pedestrian$: pure
outputs: $sigR, sigG, sigY$: pure



Reactions are occurring continually, with the continuous state variable *x* being continually updated.

When do reactions occur in a hybrid automaton?

input: $x \in \{present, absent\}$
output: $y \in \{present, absent\}$



Suppose *x* and *y* are discrete and pure signals.
When does the transition occur?

Answer: at the earliest time t when x is absent after entering s1. This will always be the same time when s1 is entered. Why?

If x is absent when s1 is entered, then the transition is taken then. If x is present when s1 is entered, then it will be absent at a time infinitesimally larger. How to model this rigorously?

Example: Newton's Cradle

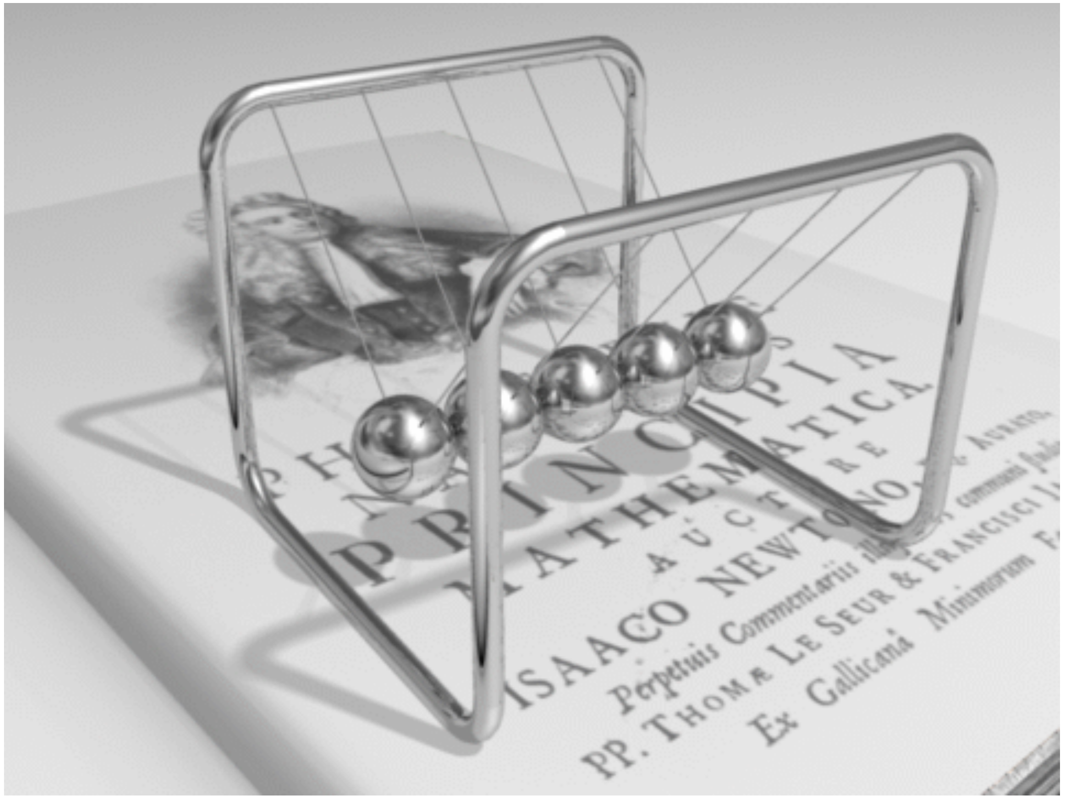


Image src: Wikipedia Commons

A middle ball does not move, so its momentum must be 0. But the momentum of the first ball is transferred somehow to the fifth. So there is an instant at which it is non-zero!

Sneak Preview: Super-Dense Time

We will solve this problem by modeling signals as functions of super-dense time:

$$x : \mathbb{R} \times \mathbb{N} \to \{present, absent\}$$

In this way, *x* can be present and absent at the same time, with a well-defined order between its presence and absence.