Sample Solutions of HW of Chapter 13: Random Numbers and Simulation

Yu-Rong Wang and Cheng-Hsin Hsu

Note that, the solutions are for your reference only. If you have any doubts about the correctness of the answers, please let the instructor and the TA know. More importantly, like other math questions, the homework questions may be solved in various ways. Do not assume that the sample solutions here are the only *correct* answers; discuss with others about alternate solutions.

We will not grade your homework assignment, but you are highly encouraged to discuss with us during the Lab hours. The correlation between the homework assignments and quiz/midterm/final questions is high. So you do want to practice more and sooner.

1 Exercises

- **13.1**

  (a) Note that $x_k = a^k$ for $k \leq 13$ and $x_k = 0$ for $k > 13$. Therefore, the period is 1 after 14 iterations.

  (b) For $a = 125$, $x_{2048} = x_0 = 1$. Therefore, the period is 2048.

  (c) The longest period is at most 8192. However, since $b = 0$ and $M = 8192$, an even value for a will generate a sequence of even numbers and an odd value for a will generate a sequence of odd numbers. For even numbers, they factor to $2n$ for some number n. After at most thirteen iterations, this becomes $2^{13}n = 8192n$ which is 0 modulo 8192. Therefore, the longest sequence must exist in the odd numbers, with a period no more than 4096. An exhaustive search of the entire domain of a yields a longest period of 2048.

- **13.2**

  (a) Well-structured sports such as baseball or football break down into a well-defined sequence of discrete steps, each with a fairly limited number of possible outcomes. A baseball game, for example, is composed of nine innings, each of which is composed of three outs for each team. A given inning is composed of a sequence of batters, each of whom faces a sequence of pitches. Thus, a baseball game can be simulated at a pitch-by-pitch level. The random choices might include the type of pitch thrown for each pitch (fastball, curve, etc.), and the resulting outcome could be determined probabilistically from a given pitcher's known effectiveness (balls

vs. strikes, walks vs. strikeouts, etc.) and each batter's known effectiveness (batting average, etc.). Similarly, football breaks down into a sequence of possessions by each team, which are further subdivided into downs, and each down offers a somewhat limited number of options (run, pass, punt, etc.). The random choices might include the type of play run on each down, and the resulting outcome could be determined probabilistically from known statistics for the quarterbacks, running backs, etc.

(b) Less well-structured sports such as basketball, soccer, and hockey do not break down so readily into a well-defined sequence of discrete steps with options and outcomes that are easily characterized. In basketball, for example, teams alternate possessions after made baskets, but unlike baseball or football, these possessions do not break down into discrete substeps, and play between baskets is relatively unstructured. Such play could still be modeled stochastically, but at a much more gross level.