# Sample Solutions of HW of Chapter 12: Fast Fourier Transform

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Note that, the solutions are for your reference only. If you have any doubts about the correctness of the answers, please let the instructor and the TA know. More importantly, like other math questions, the homework questions may be solved in various ways. Do not assume that the sample solutions here are the only *correct* answers; discuss with others about alternate solutions.

We will not grade your homework assignment, but you are highly encouraged to discuss with us during the Lab hours. The correlation between the homework assignments and quiz/midterm/final questions is high. So you do want to practice more and sooner.

## 1 Exercises

- **12.2**
  (a) $\sin(4\pi t)$
  (b) You will always obtain the same value.
  (c) To obtain an accurate value for the true frequency, the signal would need to be sampled four times per second.
- **12.5** $F_n$ is Hermitian for $n = 1$ and $n = 2$.

- **12.9**

$$\frac{1}{n}\overline{\mathrm{DFT}(\bar{\boldsymbol{y}})} = \frac{1}{n}\overline{\begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & w^1 & w^2 & \cdots & w^{n-1} \\ 1 & w^2 & w^4 & \cdots & w^{2(n-1)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & w^{n-1} & w^{2(n-1)} & \cdots & w^{(n-1)^2} \end{bmatrix}\begin{bmatrix} \bar{y}_0 \\ \bar{y}_1 \\ \bar{y}_2 \\ \vdots \\ \bar{y}_{n-1} \end{bmatrix}}$$

$$= \frac{1}{n}\begin{bmatrix} \overline{\overline{y_0}} + \overline{\overline{y_1}} + \overline{\overline{y_2}} + \cdots + \overline{\overline{y_{n-1}}} \\ \overline{\overline{y_0}} + \overline{w^1\overline{y_1}} + \overline{w^2\overline{y_2}} + \cdots + \overline{w^{n-1}\overline{y_{n-1}}} \\ \overline{\overline{y_0}} + \overline{w^2\overline{y_1}} + \overline{w^4\overline{y_2}} + \cdots + \overline{w^{2(n-1)}\overline{y_{n-1}}} \\ \vdots \\ \overline{\overline{y_0}} + \overline{w^{n-1}\overline{y_1}} + \overline{w^{2(n-1)}\overline{y_2}} + \cdots + \overline{w^{(n-1)^2}\overline{y_{n-1}}} \end{bmatrix}$$

$$= \frac{1}{n}\begin{bmatrix} y_0 + y_1 + y_2 + \cdots + y_{n-1} \\ y_0 + \overline{w^1}y_1 + \overline{w^2}y_2 + \cdots + \overline{w^{n-1}}y_{n-1} \\ y_0 + \overline{w^2}y_1 + \overline{w^4}y_2 + \cdots + \overline{w^{2(n-1)}}y_{n-1} \\ \vdots \\ y_0 + \overline{w^{n-1}}y_1 + \overline{w^{2(n-1)}}y_2 + \cdots + \overline{w^{(n-1)^2}}y_{n-1} \end{bmatrix}$$

$$= \frac{1}{n}\begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & \overline{w^1} & \overline{w^2} & \cdots & \overline{w^{n-1}} \\ 1 & \overline{w^2} & \overline{w^4} & \cdots & \overline{w^{2(n-1)}} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & \overline{w^{n-1}} & \overline{w^{2(n-1)}} & \cdots & \overline{w^{(n-1)^2}} \end{bmatrix}\begin{bmatrix} y_0 \\ y_1 \\ y_2 \\ \vdots \\ y_{n-1} \end{bmatrix} = \frac{1}{n}\boldsymbol{F}_n^H\boldsymbol{y},$$

so the stated formula indeed gives the inverse DFT.