# SOLUTION

Ex 12.1: 1, 2, 6, 13, 18

Ex 12.2: 1, 3, 5, 9, 12, 17

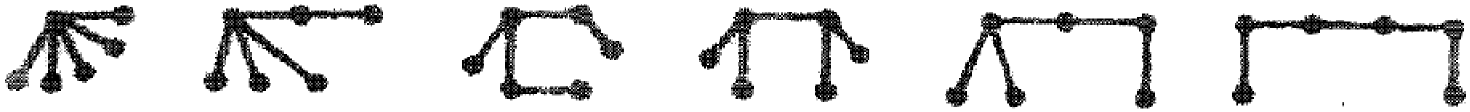
Ex 12.3: 1, 2, 3

Ex 12.4: 1, 3, 5, 7

Ex 12.5: 1, 2, 10

# Ex 12.1: (1)

a)



b) 5

# Ex 12.1: (2)

- $|E_1| = 17 \Rightarrow |V_2| = 18.$
  $|V_2| = 2|V_1| = 36 \Rightarrow |E_2| = 35.$

# Ex 12.1: (6)

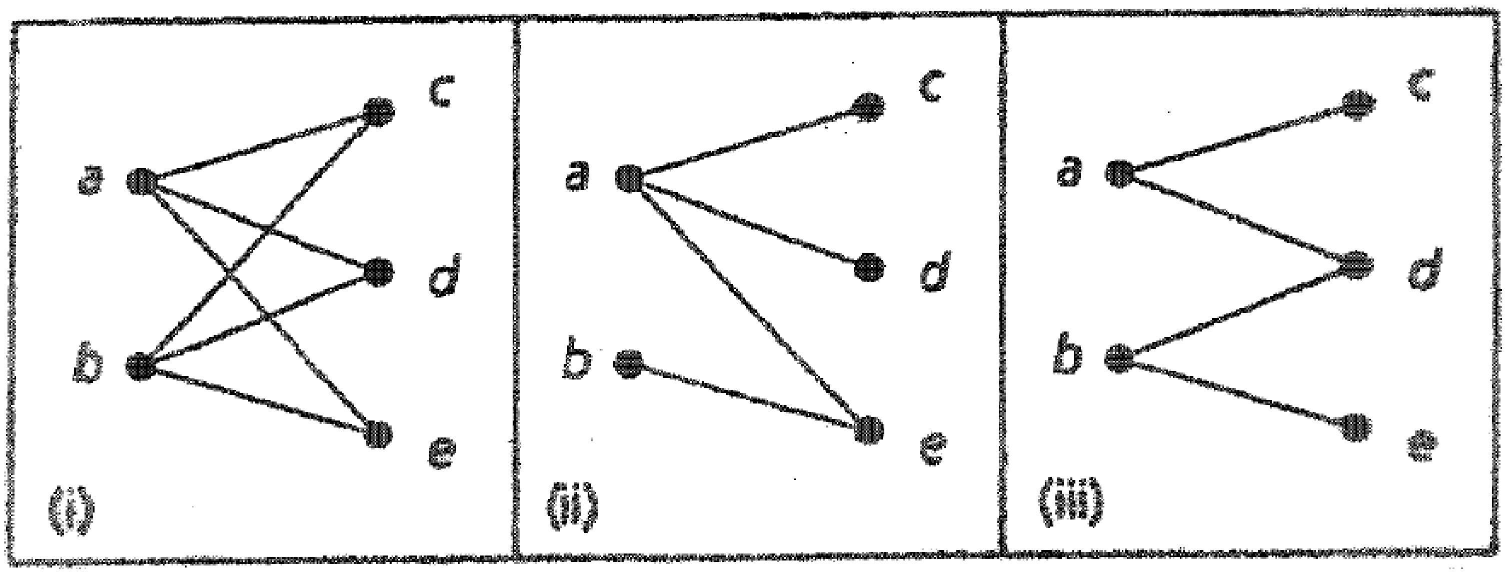
a) Since a tree contains no cycles it cannot have a subgraph homomorphic to either $K_5$ or $K_{3,3}$.

b) If $T = (V, E)$is a tree the $T$ is connected and , by part (a), $T$ is planar. By Theorem 11.6, $|V| - |E| + 1 = 2$ or $|V| = |E| + 1$.

# Ex 12.1: (13)

a) In part (i) of the given figure we find the complete bipartite graph $K_{2,3}$. Parts (ii) and (iii) of the figure provide two nonisomorphic spanning trees for $K_{2,3}$.

b) Up to isomorphism these are the only spanning trees for $K_{2,3}$.



# Ex 12.1: (18)

- $\sum_{v \in V} \deg(v) = 2|E| = 2(|V| - 1) = 2(999) = 1998.$

# Ex 12.2: (1)

a) $f, h, k, p, q, s, t$
b) $a$
c) $d$
d) $e, f, j, q, s, t$
e) $q, t$
f) 2
g) $k, p, q, s, t$

# Ex 12.2: (3)

a) $1 + w - xy * \pi \uparrow z3$

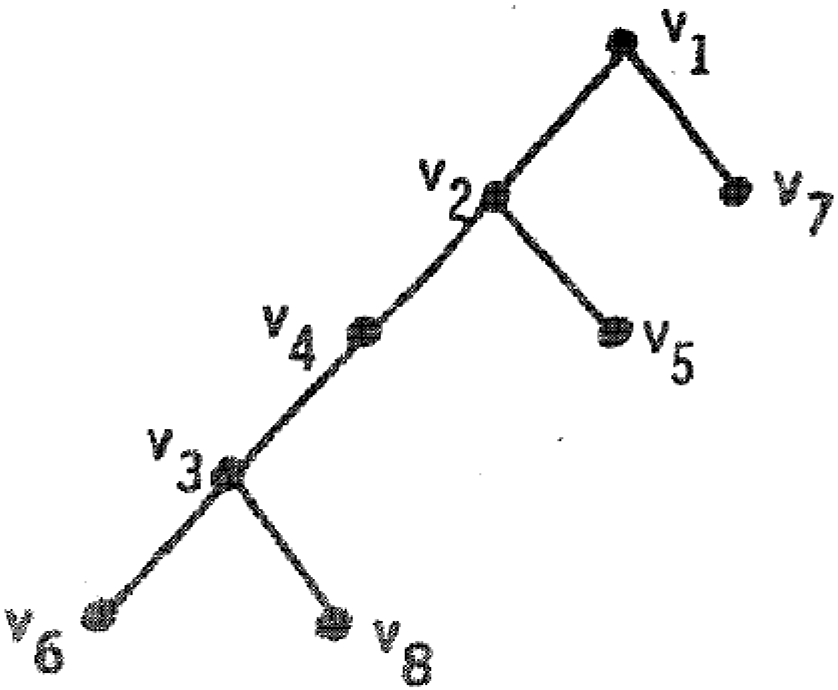
b) $0.4$

# Ex 12.2: (5)

- Preorder: r,j,h,g,e,d,b,a,c,f,i,k,m,p,s,n,q,t,v,w,u
  Inorder: h,e,a,b,d,c,g,f,j,i,r,m,s,p,k,n,v,t,w,q,u
  Psotorder: a,b,c,d,e,f,g,h,i,j,s,p,m,v,w,t,u,q,n,k,r

# Ex 12.2: (9)

- G is connected.



# Ex 12.2: (12)

- From Theorem 12.6 (c) we have

a) $\frac{l-1}{m-1} = \frac{n-1}{m} \Rightarrow (n-1)(m-1) = m(l-1)$

$\Rightarrow n - 1 = \frac{ml-m}{m-1}$

$\Rightarrow n = \left[\frac{ml-m}{m-1}\right] + 1 = \frac{[(ml-m)+(m-1)]}{m-1} = \frac{ml-1}{m-1}.$

b) $\frac{l-1}{m-1} = \frac{n-1}{m} \Rightarrow l - 1 = \frac{(m-1)(n-1)}{m}$

$\Rightarrow l = \frac{[(m-1)(n-1)+m]}{m} = \frac{[(m-1)n+1]}{m}.$

# Ex 12.2: (17)

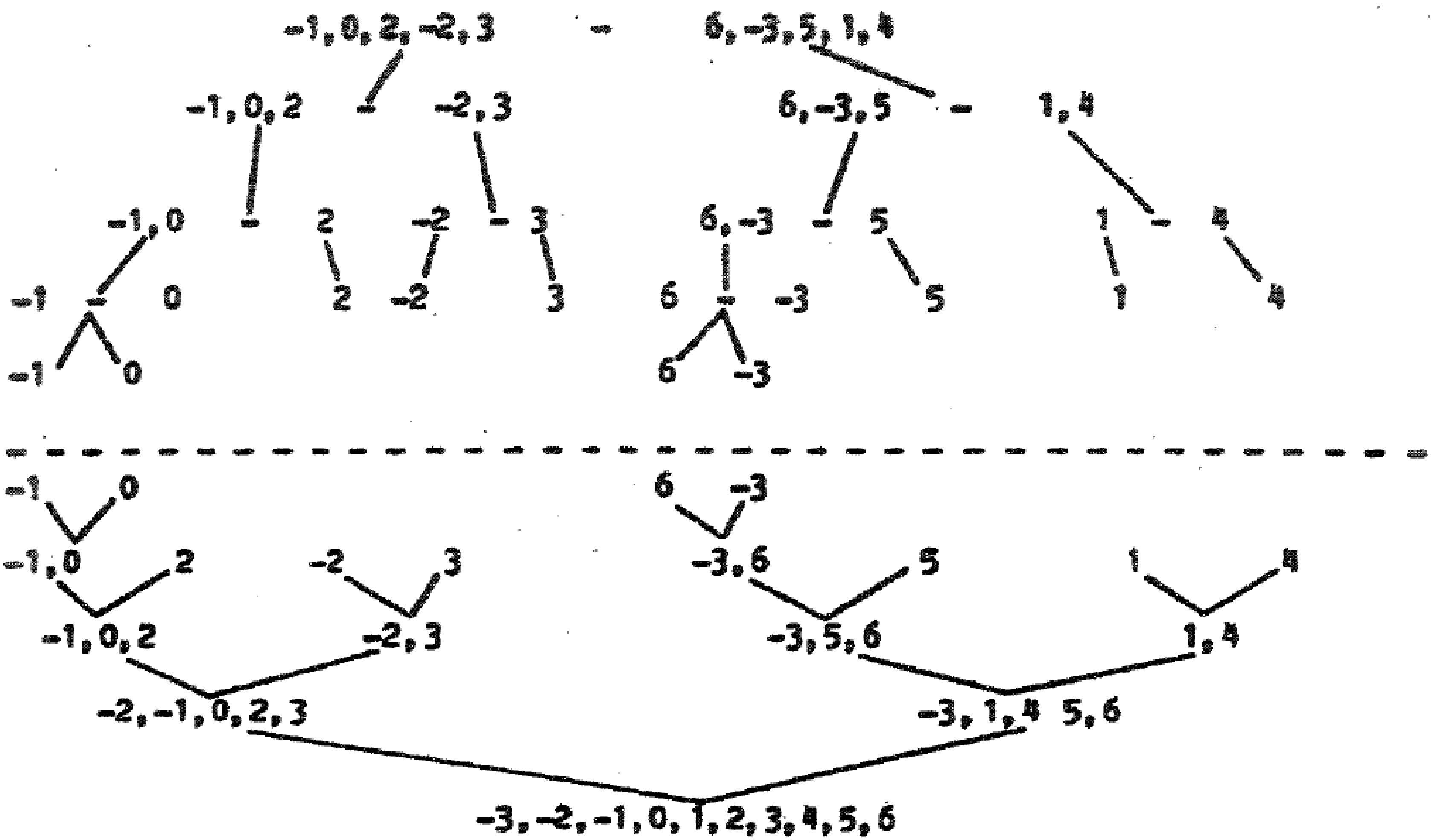
- 21845; $1 + m + m^2 + \cdots + m^{h+1} = \frac{m^h - 1}{m - 1}$.

# Ex 12.3: (1)

a) $L_1$: $1,3,5,7,9$
$L_2$: $2,4,6,8,10$

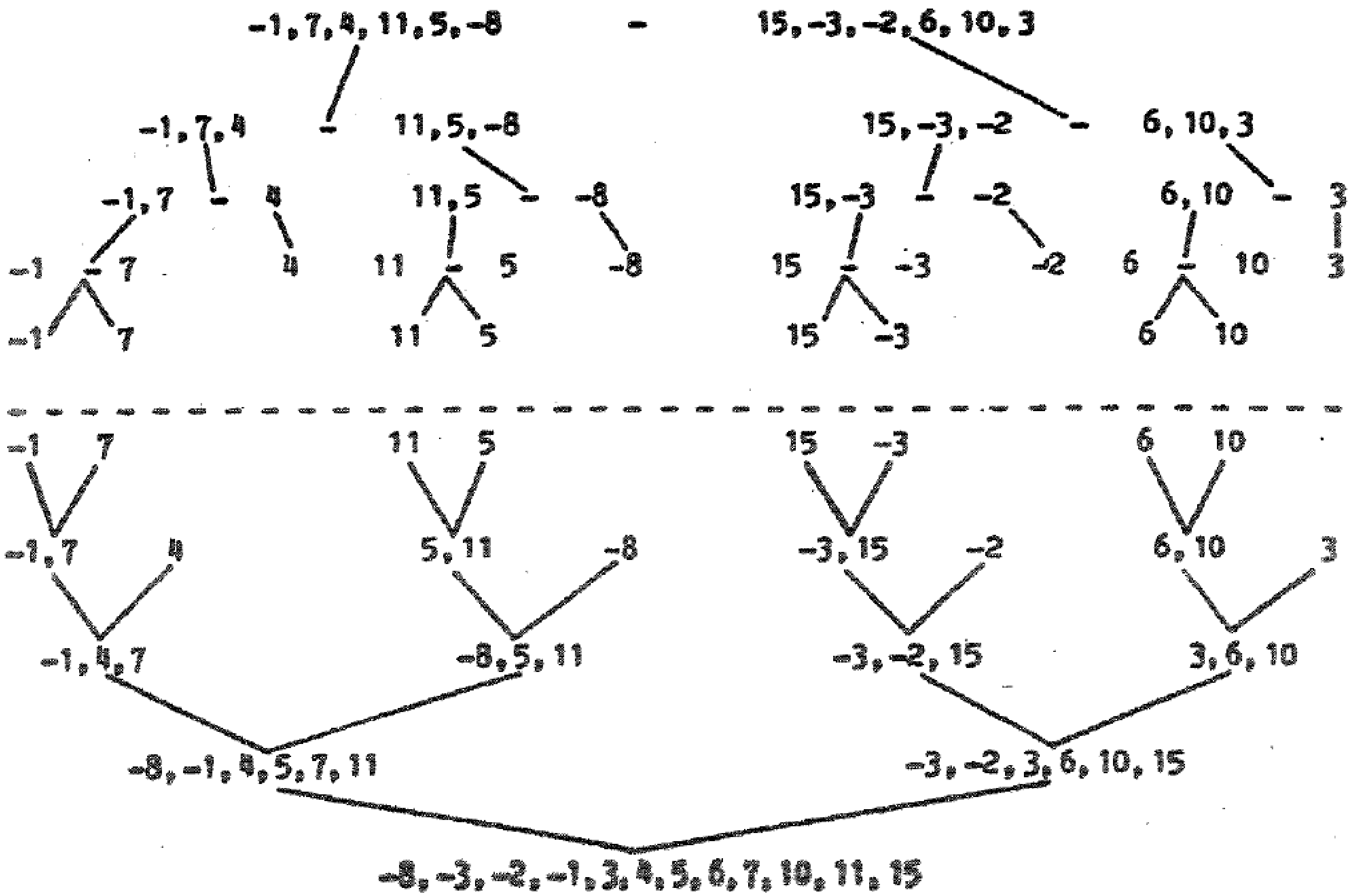
b) $L_1$: $1,3,5,7,\ldots,2m-3,m+n$
$L_2$: $2,4,6,8,\ldots,2m-2,2m-1,2m,2m+1,\ldots,m+n-1$

# Ex 12.3: (2.a)



-1, 0, 2, -2, 3
6, -3, 5, 1, 4
-1, 0, 2
-2, 3
6, -3, 5
1, 4
-1, 0
2
-2
3
6, -3
5
1
4
-1
0
2
-2
3
6
-3
5
1
4
-1
0
6
-3
-1
0
6
-3
-1, 0
2
-2
3
-3, 6
5
1
4
-1, 0, 2
-2, 3
-3, 5, 6
1, 4
-2, -1, 0, 2, 3
-3, 1, 4 5, 6
-3, -2, -1, 0, 1, 2, 3, 4, 5, 6

# Ex 12.3: (2.b)



-1, 7, 4, 11, 5, -8
-
15, -3, -2, 6, 10, 3
-1, 7, 4
-
11, 5, -8
15, -3, -2
-
6, 10, 3
-1, 7
-
4
11, 5
-
-8
15, -3
-
-2
6, 10
-
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-1
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7
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-3
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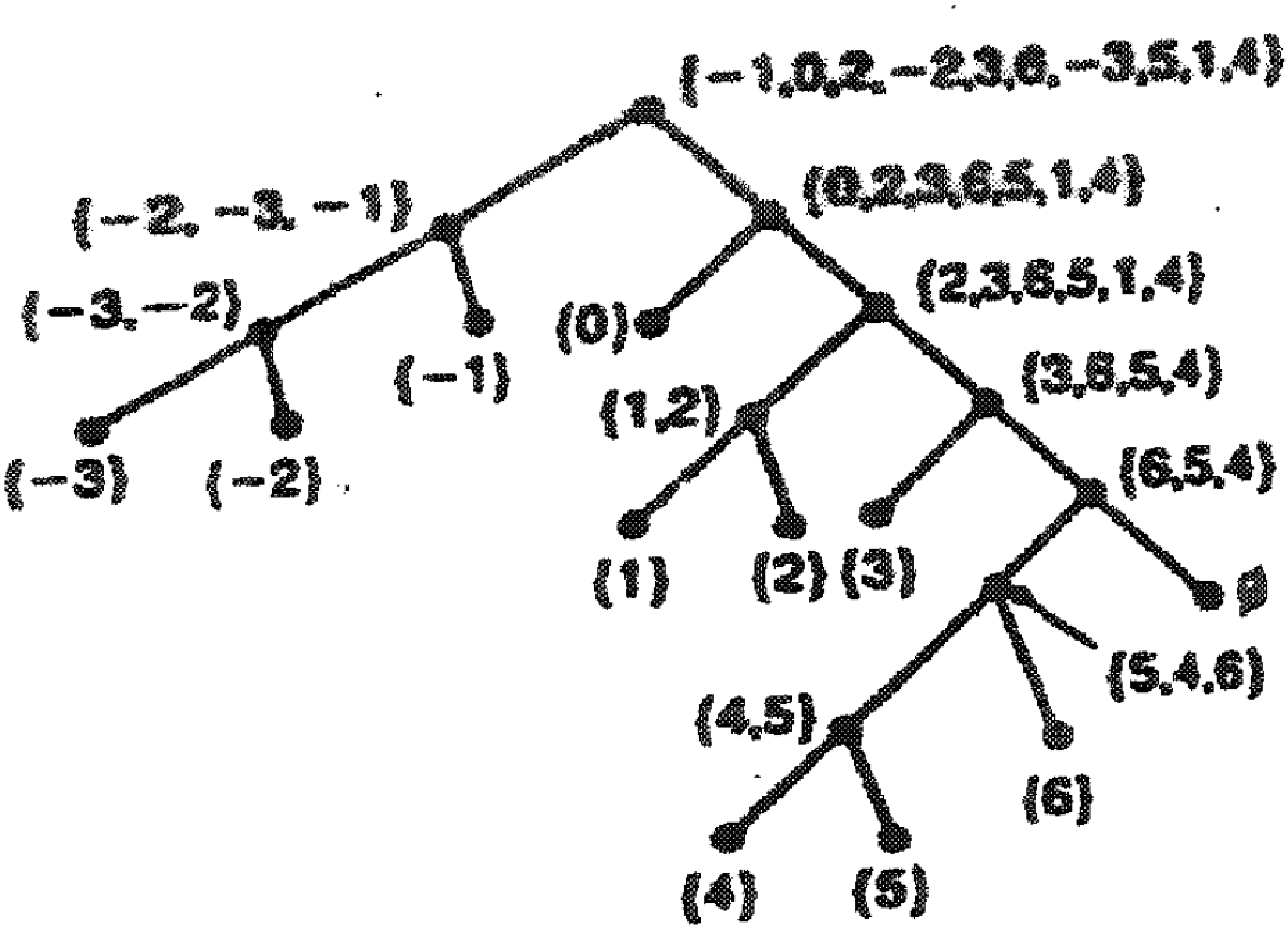
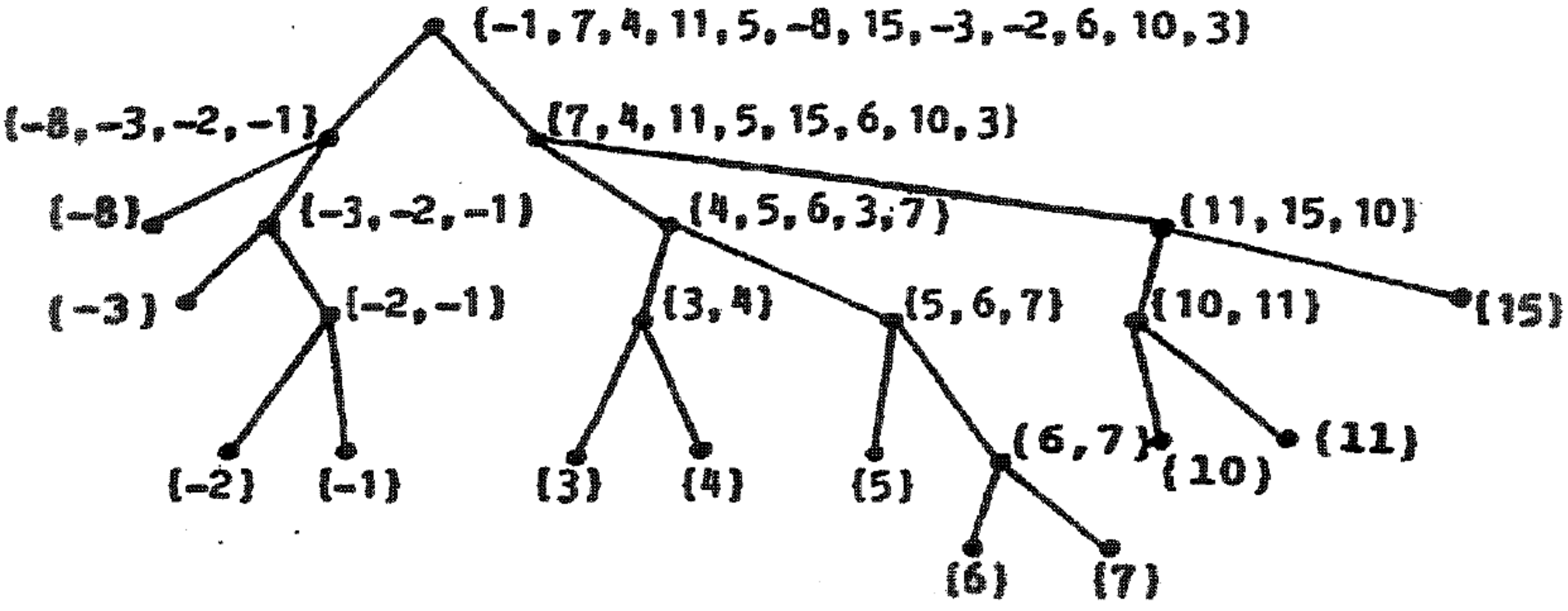
-1, 7
4
5, 11
-8
-3, 15
-2
6, 10
3
-1, 4, 7
-8, 5, 11
-3, -2, 15
3, 6, 10
-8, -1, 4, 5, 7, 11
-3, -2, 3, 6, 10, 15
-8, -3, -2, -1, 3, 4, 5, 6, 7, 10, 11, 15

# Ex 12.3: (3.a)


{−1,0,2,−2,3,6,−3,5,1,4}
{−2, −3, −1}
{0,2,3,6,5,1,4}
{−3, −2}
{−1}
{0}
{2,3,6,5,1,4}
{1,2}
{3,6,5,4}
{−3}
{−2}
{6,5,4}
{1}
{2}
{3}
∅
{5,4,6}
{4,5}
{6}
{4}
{5}

# Ex 12.3: (3.b)



{−1, 7, 4, 11, 5, −8, 15, −3, −2, 6, 10, 3}
{−8, −3, −2, −1}
{7, 4, 11, 5, 15, 6, 10, 3}
{−8}
{−3, −2, −1}
{4, 5, 6, 3, 7}
{11, 15, 10}
{−3}
{−2, −1}
{3, 4}
{5, 6, 7}
{10, 11}
{15}
{−2}
{−1}
{3}
{4}
{5}
{6, 7}
{10}
{11}
{6}
{7}

# Ex 12.4: (1)

a) tear
b) tatener
c) rant

# Ex 12.4: (3)

| | | | | | |
|---|---|---|---|---|---|
| $a$: | 111 | $e$: | 10 | $h$: | 010 |
| $b$: | 110101 | $f$: | 0111 | $i$: | 00 |
| $c$: | 0110 | $g$: | 11011 | $j$: | 110100 |
| $d$: | 0001 | | | | |

# Ex 12.4: (5)

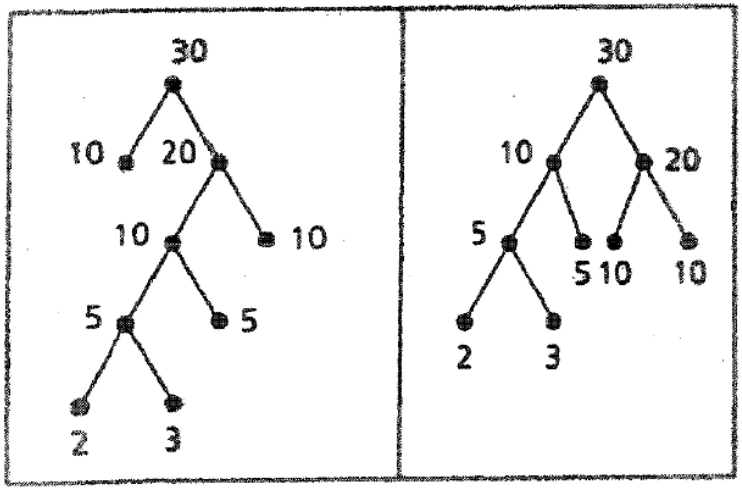
- Since the tree has $m^7 = 279{,}936$ leaves, it follows that $m = 6$. From part (c) of Theorem 12.6 we find that there are $\frac{m^7-1}{m-1} = \frac{279{,}935}{5} = 55{,}987$ internal vertices.

# Ex 12.4: (7)

- Amend part (a) of Step 2 for the Huffman tree algorithm as follows. If there are $n(> 2)$ such trees with smallest root weights $w$ and $w$’, then

(i) if $w < w$’ and $n - 1$ of these trees have root weight $w$’, select a tree (of root weight $w$’) with smallest height; and

(ii) if $w = w$’ (and all $n$ trees have the same smallest root weight), select two trees (of root weight $w$) of smallest height.



# Ex 12.5: (1)

- The articulation points are $b, e, f, h, j, k$. The biconnected components are

$$B_1\colon \{\{a,b\}\};$$
$$B_2\colon \{\{d,e\}\};$$
$$B_3\colon \{\{b,c\},\{c,f\},\{f,e\},\{e,b\}\};$$
$$B_4\colon \{\{f,g\},\{g,h\},\{h,f\}\};$$
$$B_5\colon \{\{h,i\},\{i,j\},\{j,h\}\};$$
$$B_6\colon \{\{j,k\}\};$$
$$B_7\colon \{\{k,p\},\{p,n\},\{n,m\},\{m,k\},\{p,m\}\}.$$

# Ex 12.5: (2)

- If every path from $x$ to $y$ contains the vertex $z$, then splitting the vertex $z$ will result in at least two components $C_x, C_y$ where $x \in C_x, y \in C_y$. If not, there is a path that still connects $x$ and $y$ and this path does not include vertex $z$. Conversely, if $z$ is an articulation point of $G$ then the splitting of $z$ results in at least two components $C_1, C_2$ for $G$. Select $x \in C_1, y \in C_2$. Since $G$ is connected there is at least one path from $x$ to $y$, but since $x$ and $y$ become separated upon the splitting of $z$, every path connecting $x$ and $y$ in $G$ contains the vertex $z$.

# Ex 12.5: (10)

- The ordered pair next to each vertex $v$ in the figure provides $(dfi(v), low(v))$. Following step (3) of the algorithm for determining the articulation points of $G$ we see here that this graph has four articulation points – namely, $c, e, f$, and $h$. There are five biconnected components – the figure shows the spanning trees for these components.