Name: Student ID:

Quiz #2 (4%)

CS2336 Discrete Mathematics, Instructor: Cheng-Hsin Hsu

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3:30 - 3:50 p.m., March 18th, 2013

This is a closed book test. Any academic dishonesty will automatically lead to zero point.

1) (1%) For primitive statements p, q:
   a) (0.25%) verify that $p \rightarrow [q \rightarrow (p \wedge q)]$ is a tautology.
   b) (0.25%) verify that $(p \vee q) \rightarrow [q \rightarrow q]$ is a tautology by using the results from part (a) along with the substitution rules and the laws of logic.
   c) (0.5%) is $(p \vee q) \rightarrow [q \rightarrow (p \wedge q)]$ a tautology?

Answer:

a)

p	q	$p \wedge q$	$q \rightarrow (p \wedge q)$	$p \rightarrow [q \rightarrow (p \wedge q)]$
0	0	0	1	1
0	1	0	0	1
1	0	0	1	1
1	1	1	1	1

$\Rightarrow p \rightarrow [q \rightarrow (p \wedge q)]$ is a tautology.

b) Replacing p to $(p \vee q)$ in (a).

$\Rightarrow (p \vee q) \rightarrow [q \rightarrow ((p \vee q) \wedge q)]$ (the substitution rules)

$\Rightarrow (p \vee q) \rightarrow [q \rightarrow q]$ (the absorption law)

$\Rightarrow p \rightarrow [q \rightarrow (p \wedge q)] \Leftrightarrow (p \vee q) \rightarrow [q \rightarrow q]$ is a tautology.

c)

p	q	$p \vee q$	$p \wedge q$	$q \rightarrow (p \wedge q)$	$(p \vee q) \rightarrow [q \rightarrow (p \wedge q)]$
0	0	0	0	1	1
0	1	1	0	0	0
1	0	1	0	1	1
1	1	1	1	1	1

$\Rightarrow (p \vee q) \rightarrow [q \rightarrow (p \wedge q)]$ is not a tautology.

2) (1%) Write each of the following arguments in symbolic form. Then establish the validity of the argument or give a counter example to show that it is invalid.

a) If Rochelle gets the supervisor's position and works hard, then she will get a raise. If she gets the raise, then she will buy a new car. She has not purchased a new car. Therefore, either Rochelle did not get the supervisor's position or she did not work hard.

b) If there is a chance of rain or her red headband is missing, then Lois will not mow her lawn. Whenever the temperature is over 80 ° F, there is no chance for rain. Today, the temperature is 85 ° and Lois is wearing her red headband. Therefore Lois will mow her lawn sometime today.

Answer:

p: Rochelle gets the supervisor's position. r: Rochelle gets the raise.

q: Rochelle works hard. s: Rochelle buy a new car.

a)

1) $\neg s$	Premise
2) $r \rightarrow s$	Premise
3) $\neg r$	Steps (1), (2) and Modus Tollens
4) $(p \wedge q) \rightarrow r$	Premise
5) $\neg(p \wedge q)$	Steps (3), (4) and Modus Tollens
6) $\neg p \vee \neg q$	Step (5) and $\neg(p \wedge q) \leftrightarrow \neg p \vee \neg q$

b)

p: There is a chance of rain.	r: Lois does not mow her lawn.
q: Lois' red headband is missing	s: The temperature is over 80^o F.
1) $s \to \neg p$	Premise
2) $s \wedge \neg q$	Premise
3) $\neg p$	Step (1), (2) and Rule of Detachment
4) $(p \vee q) \to r$	Premise
5) $\neg r \to (\neg p \wedge \neg q)$	Negation the step (4)
6) $\neg r$	

3) (1%) Let $p(x, y) : x^2 \geq y$ and $q(x, y) : x + 2 > y$ be two open statements. Consider a universe of all real numbers, determine the truth value for each of the following statements.

a) $p(2, 4)$

b) $q(1, \pi)$

c) $p(-3, 8) \wedge q(1, 3)$

d) $p(1, 2) \leftrightarrow \neg q(1, 2)$

e) $p(2, 2) \to q(1, 1)$

Answer:

a) True, $2^2 \geq 4$

b) False, $1 + 2 > \pi$

c) False, $((-3)^2 \geq 8) \wedge (1 + 2 > 3) = (True \wedge False)$

d) True, $((1^2 \geq 2) \leftrightarrow \neg(1 + 2 \geq 2)) = (False \leftrightarrow False)$

e) True, $((2^2 \geq 2) \to (1 + 2 > 1)) = (True \to True)$

4) (1%) Let n be an integer. Prove that n is even if and only if $29n + 14$ is even.

If n is even, then $n = 2k$ for some (particular) integer k. Then $29n + 14 = 29(2k) + 14 = 2(29k + 7)$, so it follows from Definition 2.8 that $29n + 14$ is even.

Conversely, suppose that n is not even. Then n is odd, so $n = 2t + 1$ for some (particular) integer t. Therefore, $29n + 14 = 29(2t + 1) + 14 = 2(29t + 21) + 1$, so from Definition 2.8 we have $29n + 14$ odd $\cdots$ hence, not even.

Consequently, the converse follows by contraposition.