

Name:

Student ID:

Quiz #1 (4%)

CS2336 Discrete Mathematics, Instructor: Cheng-Hsin Hsu

Department of Computing Science, National Tsing Hua University, Taiwan

3:20 - 3:45 p.m., March 11th, 2013

This is a closed book test. Any academic dishonesty will automatically lead to zero point.

- 1) (1%) How many positive integers n can we form using the digits 3, 4, 4, 5, 5, 6, 7 if we want n to exceed 6,000,000?

Answer:

Case1: the leading digit is 6: $\frac{6!}{2!2!}$.

Case2: the leading digit is 7: $\frac{6!}{2!2!}$.

In total there are $2 \times \frac{6!}{2!2!} = 360$

- 2) (1%) Answer the following questions:

- In how many possible ways could a student answer a 9-question true-false test?
- In how many ways can the student answer the above test if he/she can leave a question unanswered to avoid extra penalty for wrong answers?

Answer:

- With two choices per question. There are 2^9 .
- With three choices per question. There are 3^9 .

3) (1%) Determine the coefficient of

a) xyz^2 in $(x + y + 2z)^5$

b) $w^3x^2yz^2$ in $(2w - x + 3y - 2z)^8$

Answer:

a) The coefficient of xyz^2 in $(x + y + 2z)^5$ is 0. Because xyz^2 doesn't exist.

b) The coefficient of $w^3x^2yz^2$ in $(2w - x + 3y - 2z)^8$ is

$$\binom{8}{3} \binom{5}{2} \binom{3}{1} \binom{2}{2} 2^3 (-1)^2 3 (-2)^2 = 161280.$$

4) (1%) How many different ways are there to place 16 marbles of the same size in 6 distinct jars if

a) the marbles are all blue?

b) each marble is in a different color?

Answer:

a) The number of solutions equals the number of solutions to

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 16 \text{ where } x_i \geq 0 \text{ for } 1 \leq i \leq 6.$$

$$\text{The number is } \binom{16+6-1}{16} = \binom{21}{16} = 20349.$$

b) Each marble has 6 choices. The number is 6^{16} .