Name: Student ID:

Quiz #7 6%

CS2336 Discrete Mathematics, Instructor: Cheng-Hsin Hsu

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This is a closed book test. Any academic dishonesty will automatically lead to zero point. If the total points are more than 6 points, you will get at most 6 points out of it.

1) (2%) If $A = \{w, x, y, z\}$, determine the number of relations on A that are

   a) symmetric
   b) reflexive and symmetric
   c) antisymmetric and contain (x, y)
   d) reflexive, symmetric, and antisymmetric

   Answer:

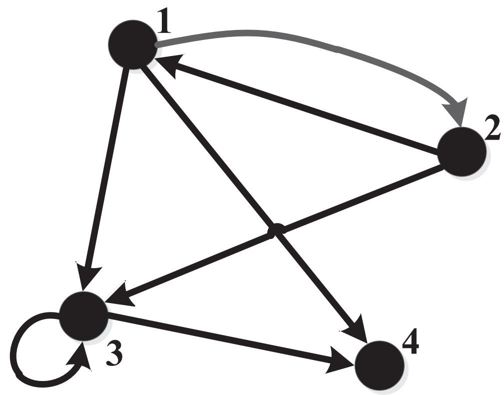
   a) 2^{10}
   b) 2^{6}
   c) $2^{4} \cdot 3^{5}$
   d) 1

2) (2%) Let $A = \{1, 2, 3, 4\}, B = \{w, x, y, z\}$. Define the relation $\mathscr{R}_1 \subseteq A \times B, \mathscr{R}_2 \subseteq B \times A$ and $\mathscr{R}_3 \subseteq B \times A$, where $\mathscr{R}_1 = \{(1, w), (3, w), (2, x), (1, y)\}$, $\mathscr{R}_2 = \{(w, 4), (x, 1), (x, 3), (y, 2)\}$ and $\mathscr{R}_3 = \{(w, 3), (y, 4)\}$.

   a) Determine $\mathscr{R}_1 \circ (\mathscr{R}_2 \cup \mathscr{R}_3)$
   b) Draw the digraph of (a)

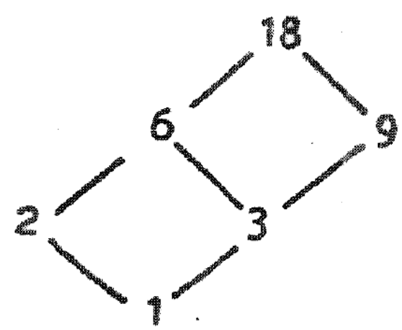
   Answer:

   a) $\mathscr{R}_1 \circ (\mathscr{R}_2 \cup \mathscr{R}_3) = \{(1, 2), (1, 3), (1, 4), (2, 1), (2, 3), (3, 3), (3, 4)\}$

b) 

3) (2%) Let $A = \{1, 2, 3, 6, 9, 18\}$, and define $\mathscr{R}$ on A by $x\mathscr{R}y$ if $x \mid y$. Draw the Hasse diagram for the poset $(A, \mathscr{R})$.

Answer:



4) (2%) If $A = \{1, 2, 3, 4, 5, 6, 7\}$, define $\mathscr{R}$ on A by $(x, y) \in \mathscr{R}$ if $x - y$ is a multiple of 3.

a) Show that $\mathscr{R}$ is an equivalence relation on A.

b) Determine the equivalence classes and partition of A induced by $\mathscr{R}$.

Answer:

a) For all $a \in A$, $a - a = 3 \cdot 0$, so $\mathscr{R}$ is reflexive. For $a, b \in A, a - b = 3c$, for some $c \in \mathbb{Z} \Rightarrow b - a = 3(-c)$, for $-c \in \mathbb{Z}$, so $a\mathscr{R}b \Rightarrow b\mathscr{R}a$ and $\mathscr{R}$ is symmetric. If $a, b, c \in A$ and $a\mathscr{R}b, b\mathscr{R}c$, then $a - b = 3m, b - c = 3n$, for some $m, n \in \mathbb{Z} \Rightarrow (a - b) + (b - c) = 3m + 3n \Rightarrow a - c = 3(m + n)$, so $a\mathscr{R}c$. Consequently, $\mathscr{R}$ is transitive.

b) $[1] = [4] = [7] = \{1, 4, 7\}; [2] = [5] = \{2, 5\}; [3] = [6] = \{3, 6\}.$

$A = \{1, 4, 7\} \cup \{2, 5\} \cup \{3, 6\}.$

5) (2%) For the finite state machine given in the state table below, determine a minimal machine that is equivalent to it. Please give the state table and the intermediate partitions as the answer (e.g., P_1, P_2...).

	ν		ω	
	0	1	0	1
s_1	s_7	s_6	1	0
s_2	s_7	s_7	0	0
s_3	s_7	s_2	1	0
s_4	s_2	s_3	0	0
s_5	s_3	s_7	0	0
s_6	s_4	s_1	0	0
s_7	s_3	s_5	1	0
s_8	s_7	s_3	0	0

Answer:

$P_1 : \{S_1, S_3, S_7\}, \{S_2, S_4, S_5, S_6, S_8\}$

$P_2 : \{S_1, S_3, S_7\}, \{S_2, S_5, S_8\}, \{S_4, S_6\}$

$P_3 : \{S_1\}, \{S_3, S_7\}, \{S_2, S_5, S_8\}, \{S_4\}, \{S_6\}$

$P_4 = P_3$

	ν		ω	
	0	1	0	1
s_1	s_3	s_6	1	0
s_2	s_3	s_3	0	0
s_3	s_3	s_2	1	0
s_4	s_2	s_3	0	0
s_6	s_4	s_1	0	0