Name: Student ID:

Quiz #2 6%

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This is a closed book test. Any academic dishonesty will automatically lead to zero point.

1) (1%) Give the reasons for each step in the following simplifications of compound statements.

   a) $(p \rightarrow q) \wedge [\neg q \wedge (r \vee \neg q)] \Leftrightarrow \neg(q \vee p)$

Answer:

$(p \rightarrow q) \wedge [\neg q \wedge (r \vee \neg q)]$

$\Leftrightarrow (p \rightarrow q) \wedge \neg q$ *reason: Absorption Law (and the Commutative Law of* $\vee$*)*

$\Leftrightarrow (\neg p \vee q) \wedge \neg q$ *reason:* $p \rightarrow q \Leftrightarrow \neg p \vee q$

$\Leftrightarrow \neg q \wedge (\neg p \vee q)$ *reason: Commutative Law of* $\wedge$

$\Leftrightarrow (\neg q \wedge \neg p) \vee (\neg q \wedge q)$ *reason: Distributive Law of* $\wedge$ *over* $\vee$

$\Leftrightarrow (\neg q \wedge \neg p) \vee F_0$ *reason: Inverse Law*

$\Leftrightarrow \neg q \wedge \neg p$ *reason: Identity Law*

$\Leftrightarrow \neg(q \vee p)$ *reason: DeMorgan's Laws*

2) *(1%) Given the reasons(s) for each step needed to show that the following arguments is valid.*

$[p \wedge (p \rightarrow q) \wedge (s \vee r) \wedge (r \rightarrow \neg q)] \rightarrow (s \vee t)$

Answer:

Steps

1) p reason: premise

2) $p \rightarrow q$ reason: premise

3) q reason: 1), 2) and the Rule of Detachment

$4)$ $r \to \neg q$ reason: premise

$5)$ $q \to \neg r$ reason: $4)$ and $(r \to \neg q) \Leftrightarrow (\neg\neg q \to \neg r) \Leftrightarrow (q \to \neg r)$

$6)$ $\neg r$ reason: $3)$, $5)$ and the Rule of Detachment

$7)$ $s \vee r$ reason: premise

$8)$ s reason: $6)$, $7)$ and the Rule of Disjunctive Syllogism

$9)$ $\therefore s \vee t$ reason: $8)$ and the Rule of Disjunctive Amplification

3) (1%) Use truth tables to verify that each of the following is a logical implication

a) $[(p \to q) \wedge (q \to r)] \to (p \to r)$

b) $[(p \vee q) \wedge \neg p] \to q$

Answer:

TABLE I

(A)

p	q	r	$p \to q$	$q \to r$	$p \to r$	$[(p \to q) \wedge (q \to r)] \to (p \to r)$
0	0	0	1	1	1	1
0	0	1	1	1	1	1
0	1	0	1	0	1	1
0	1	1	1	1	1	1
1	0	0	0	1	0	1
1	0	1	0	1	1	1
1	1	0	1	0	0	1
1	1	1	1	1	1	1

TABLE II

(B)

p	q	$\neg p$	$p \vee q$	$(p \vee q) \wedge \neg p$	$[(p \vee q) \wedge \neg p] \rightarrow q$
0	0	1	0	0	1
0	1	1	1	1	1
1	0	0	1	0	1
1	1	0	1	0	1

4) *(2%) Let* $p(x), q(x) and r(x)$ *denote the following open statements.*

$p(x) : x^2 - 8x + 15 = 0$

$q(x) : x$ *is odd*

For the universe of all integers, determine the truth or falsity of each of the following statements. If a statement is false, give a counterexample.

a) $\forall x[q(x) \rightarrow p(x)]$
b) $\exists x[q(x) \rightarrow p(x)]$
c) $\forall x[p(x) \rightarrow q(x)]$
d) $\exists x[p(x) \rightarrow q(x)]$

Answer:

a) False, For $x = 1$, $q(x)$ is true while $p(x)$ is false.
b) True
c) True
d) True

5) (1%) Let n be an integer. Prove that n is odd if and only if $7n + 8$ is odd.

Answer:

If n *is odd, then* $n = 2k + 1$ *for some (particular) integer* k*.*

Then $7n + 8 = 7(2k + 1) + 8 = 14k + 15 = 14k + 14 + 1 = 2(7k + 7) + 1$*,*

so it follows from Definition 2.8 that $7n + 8$ *is odd.*

Conversely, suppose that n *is not odd. Then* n *is even,*

so $n = 2t$ *for some (particular) integer* t*.*

But then $7n + 8 = 7(2t) + 8 = 14t + 8 = 2(7t + 4)$*,*

so it follows from Definition 2.8 that $7n + 8$ *is even – that is,*

$7n + 8$ *is not odd. Consequently, the converse follows by contraposition.*

Definition 2.8 : Let n be an integer. We call n even if n is divisible by 2 – that is if there exists an integer r so that $n = 2r$. If n is not even, then we call n odd and find for this case that there exists an integer s where $n = 2s + 1$