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Quiz #8 (5%)

CS2336 Discrete Mathematics, Instructor: Cheng-Hsin Hsu

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2:20 - 2:40 p.m., May 22nd, 2014

This is a closed book test. Any academic dishonesty will automatically lead to zero point.

1) (1%) Determine the number of integer solutions of $a + b + c + d = 19$, where $-5 \leq a, b, c, d \geq 10$.

Solution: The number of integer solutions for $a + b + c + d = 19$, $-5 \leq a, b, c, d \geq 10$, equals to The number of integer solutions for $y_1 + y_2 + y_3 + y_4 = 39$, $0 \leq y_i \geq 15$, for $1 \leq i \geq 4$. Let $c_i \geq 16$

$N(c_i), y_1 + y_2 + y_3 + y_4 = 23$: $\binom{4+23-1}{23} = \binom{26}{23}$

$N(c_i, c_j), y_1 + y_2 + y_3 + y_4 = 7$: $\binom{4+7-1}{7} = \binom{10}{7}$

$N(\overline{c_1}, \overline{c_2}, \overline{c_3}, \overline{c_4}) = \binom{42}{39} - 4\binom{26}{23} + 6\binom{10}{7}$

2) (2%)

a) In how many ways can the letters in ARRANGEMENT be arranged so that there are exactly two pairs of consecutive identical letters?

b) Answer part (a), but with at least two pairs of consecutive identical letters?

Solution: Let c_1 be the condition of the two A's are together in an arrangement of ARRANGEMENT. Conditions c_2, c_3, c_4 are defined similarly for the two E's, N's and R's, respectively.

$N = \frac{11!}{(2!)^4} = 2494800$

For $1 \leq i \geq 4, N(c_i) = \frac{10!}{(2!)^3} = 453600$

For $1 \leq i, j \geq 4, N(c_i, c_j) = \frac{9!}{(2!)^2} = 90720$

For $1 \leq i, j, k \geq 4, N(c_i, c_j, c_k) = \frac{8!}{(2!)} = 20160$

$N(c_1, c_2, c_3, c_4) = 7! = 5040$

let $S_1 = \binom{4}{1}\frac{10!}{(2!)^3}$, $S_2 = \binom{4}{2}\frac{9!}{(2!)^2}$, $S_3 = \binom{4}{3}\frac{8!}{(2!)}$, and $S_4 = \binom{4}{4}7!$

a) $E_2 = S_2 - \binom{3}{1}S_3 + \binom{4}{2}S_4 = 332640$

b) $L_2 = S_2 - \binom{2}{1}S_3 + \binom{3}{1}S_4 = 398160$

3) (2%) Find the rook polynomials for the shaded chessboards in Fig. 1.

i) $1 + 6x + 12x^2 + 8x^3$

ii) $1 + 8x + 14x^2 + 4x^3$

iii) $1 + 9x + 25x^2 + 21x^3$

iv) $1 + 8x + 16x^2 + 7x^3$

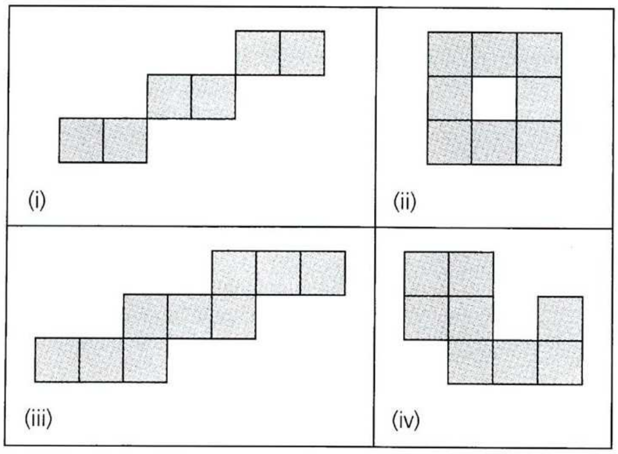

Fig. 1. The chessboards.