Name: Student ID:

Quiz #3 (5% + 1% Bonus Point)

CS2336 Discrete Mathematics, Instructor: Cheng-Hsin Hsu

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3:30 - 3:50 p.m., March 31st, 2013

This is a closed book test. Any academic dishonesty will automatically lead to zero point.

1) (2%) Determine $|A \cup B \cup C|$, when $|A| = 10$, $|B| = 100$, and $|C| = 1000$ and
   a) (0.5%) $A \cap B = A \cap C = B \cap C = \emptyset$.
   b) (0.5%) $A \subseteq B \subseteq C$.
   c) (1%) $|A \cap B| = |A \cap C| = |B \cap C| = 3$ and $|A \cap B \cap C| = 1$.

   Solution:

   $|A \cup B \cup C| = |A| + |B| + |C| - (|A \cap B| + |B \cap C| + |A \cap C|) + |A \cap B \cap C|$
   a) $|A \cup B \cup C| = (10 + 100 + 1000) = 1110$
   b) $|A \cup B \cup C| = (10 + 100 + 1000) - (10 + 100 + 10) + 10 = 1000$
   c) $|A \cup B \cup C| = (10 + 100 + 1000) - (3 + 3 + 3) + 1 = 1102$

2) (2%) Randomly choose 3 different numbers from $\{1, 2, 3, 4, \ldots, 20\}$, what is the probability their sum is even?

   Solution:

   There are 2 cases when the sum of 3 different numbers is even:

   (1) All 3 numbers are even. There are $\binom{10}{3} = 120$ ways

   (2) 1 chosen number is even, and 2 are odd. There are $\binom{10}{2}\binom{10}{1} = 450$ ways

   Therefore, the probability is $\frac{120+450}{\binom{20}{3}} = \frac{1}{2}$

3) (2%) Prove or disprove:

   a) For sets $A, B, C \subseteq \mathbb{Z}, A \cap C = B \cap C \Rightarrow A = B$.

   b) For sets $A, B, C \subseteq \mathbb{Z}^+, [(A \cap C = B \cap C) \wedge (A \cup C = B \cup C)] \Rightarrow A = B$.

   Solution:

   a) Let $A = \{1, 2\}, B = \{1, 3\}, C = \{1\}$. Then $A \cap C = B \cap C$, but $A \neq B$.

   b) Let $x \in A \Rightarrow x \in A \cup C = B \cup C$. So $x \in B$ or $x \in C$.

      If $x \in C$, then $x \in A \cap C = B \cap C$, so $x \in B$.

      In either case, $x \in B$, so $A \subseteq B$.

      Likewise, let $y \in B \Rightarrow y \in B \cup C = A \cup C$. So $y \in A$ or $y \in C$.

      If $y \in C$, then $y \in B \cap C = A \cap C$, so $y \in A$.

      In either case, $y \in A$, so $B \subseteq A$.

      Since $A \subseteq B$ and $B \subseteq A$, therefore $A = B$.