Name: Student ID:

# Quiz #2 (5% + 1% Bonus)

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3:30 - 3:50 p.m., March 17th, 2014

**This is a closed book test. Any academic dishonesty will automatically lead to zero point.**

1) (2%) For primitive statements $p$, $q$:
   a) Verify that $p \rightarrow [q \rightarrow (p \wedge q)]$ is a tautology.
   b) Verify that $(p \vee q) \rightarrow [q \rightarrow q]$ is a tautology by using the results from part (a) along with the substitution rules and the laws of logic.
   c) Is $(p \vee q) \rightarrow [q \rightarrow (p \wedge q)]$ a tautology?

*Solution:*

(a)

| $p$ | $q$ | $p \wedge q$ | $q \rightarrow (p \wedge q)$ | $p \rightarrow [q \rightarrow (p \wedge q)]$ |
|---|---|---|---|---|
| 0 | 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 |

$\Rightarrow p \rightarrow [q \rightarrow (p \wedge q)]$ is a tautology

*(b)*

*Replacing* $p$ *to* $(p \vee q)$ *in (a)*

$\Rightarrow (p \vee q) \rightarrow [q \rightarrow ((p \vee q) \wedge q)]$ *(the substitution rules)*

$\Rightarrow (p \vee q) \rightarrow [q \rightarrow q]$ *(the absorption law)*

$\Rightarrow p \rightarrow [q \rightarrow (p \wedge q)] \Leftrightarrow (p \vee q) \rightarrow [q \rightarrow q]$ *is a tautology.*

(c)

| $p$ | $q$ | $p \wedge q$ | $p \vee q$ | $q \to (p \wedge q)$ | $(p \vee q) \to [q \to (p \wedge q)]$ |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 1 | 1 |
| 0 | 1 | 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |

$\Rightarrow (p \vee q) \to [q \to (p \wedge q)]$ is not a tautology

2) (2%) Let $p(x, y) : x^2 \geq y$ and $q(x, y) : x + 2 > y$ be two open statements. Consider a universe of all real numbers, determine the truth value for each of the following statements.

a) $q(1, \pi)$

b) $p(-3, 8) \wedge q(1, 3)$

c) $p(1, 2) \leftrightarrow \neg q(1, 2)$

d) $p(2, 2) \to q(1, 1)$

*Solution:*

(a) False, $1 + 2 > \pi$

*(b) False,* $((3)2 \geq 8) \wedge (1 + 2 > 3) = (True \wedge False)$

(c) True, $((122) \leftrightarrow \neg(1 + 22)) = (False \leftrightarrow False)$

*(d) True,* $((222) \to (1 + 2 > 1)) = (True \to True)$

3) *(2%) Let $n$ be an integer. Prove that $n$ is even if and only if $29n + 14$ is even.*

Solution:

If $n$ is even, then $n = 2k$ for some (particular) integer $k$. Then $29n + 14 = 29(2k) + 14 = 2(29k + 7)$, so it follows from Definition 2.8 that $29n + 14$ is even. Conversely, suppose that $n$ is not even. Then $n$ is odd, so $n = 2t + 1$ for some (particular) integer $t$. Therefore, $29n + 14 = 29(2t + 1) + 14 = 2(29t + 21) + 1$, so from Definition 2.8 we have $29n + 14$ odd    hence, not even. Consequently, the converse follows by contraposition.